Digital to Analog and Analog to Digital Conversions

Basic Terms and Concepts

Digital to Analog Converters

Digital to analog converters (DAC's) convert a digital number to an analog voltage. For an 8 bit number we represent as {D7 – D0}, where D0 is the least significant bit, i.e. the number of 2^0's and D7 is the most significant number, i.e. the number of 2^7's, the analog output voltage should be given by

$$V_{out} = V_{fs}\left(\frac{D7}{2} + \frac{D6}{2^2} + \frac{D5}{2^3} + \ldots + \frac{D0}{2^8}\right) \qquad 1.$$

Here the D's are either 0 (low) or 1 (high) and V_{fs} is called the full scale voltage. The output, i.e. the analog voltage, will be between 0V and $V_{full\ scale} = V_{fs}$. (If you want a bipolar output, you need to subtract $V_{fs}/2$ from the voltage given above.) For an n-bit converter with a digital input of D_{in}, the output voltage should be

$$D_{in} \times \frac{V_{fs}}{2^n} \qquad 2.$$

For instance if the digital input for a 12-bit converter with a V_{fs} = 10.24V is 1400_{10}, V_{out} should be $1400\times10.24V/2^{12}$ = 3.500V. (Of course, the input is actually presented to the converter as a binary number, not a decimal number, or 0101 0111 1000. However, it is easier to do the calculations in decimal.)

The basic terms are

1. **Resolution**. This is the number of bits in the digital number to be converted. Common sizes are 8 bit, 10 bit, 12 bit, 14 bit and 16 bit converters. (You often see 18, 20 or even 24 bit converters in precision applications.) An 8 bit digital number can represent the decimal values of 0 through 255, or 2^8 =256 values, a 16-bit number can represent the numbers 0 through 2^{16}-1, or 0 though 65,535.
2. **Full Scale Voltage**. This is the "maximum" voltage output, often V_{fs}. For an n bit converter the output voltage typically ranges from 0V to $\frac{2^n-1}{2^n}V_{fs}$. For an 8 bit converter and V_{fs} = 10Vthe range of output voltages would be 0 to 9.961V.
3. **Step Size**. This is how much the output voltage would change when the digital number changes by 1 LSB (least significant bit) or by 1 bit. (This voltage change is sometimes called a change of 1 LSB.) This is $\frac{V_{fs}}{2^n}$. For an 8 bit converter and a full scale voltage of 10 volts this is 0.039V or 39mV. Note that the maximum output is really the full scale voltage minus the step size.
4. **Accuracy**. This term represents how close the actual output voltage comes to the theoretical output voltage given by equation 1 above. For a 10bit converter and a V_{fs} = 10.00V, and the binary number 00 1000 0000 = 128_{10}, the output voltage should be 128×step size = $128\times10V/2^{10}$ = 1.250V. If the actual output is 1.240V, the error is 0.010V. This is usually specified in terms of bits (basically how many step sizes the error is). Here the step size is 10.00V/1024 = 0.0098V ≈ 0.010V. Therefore the error is about 1 step size, or one LSB. If the error was 0.020V, the error would be two bits. If the error was 0.005V it would be ½ bit. If the accuracy is ± ½ bit, then you are guaranteed that an increase in the digital number will not result in a decrease in the analog output voltage. (You usually look for accuracies of ± ½ bit when selecting a DAC.)

5. **Linearity**. This term refers to how close the output voltages are to a "best fit" straight line to the analog output voltages, i.e. when V_{out} is plotted vs. the digital value each one represents. This is a popular term with manufacturers, since it makes their DAC's seem slightly better than they really are – compared to specifying the accuracy.

Analog to Digital Converters

Analog to digital converters, ADC's, convert an analog input voltage, V_{in}, to a digital number. All of the terms above also apply to ADC's. An ADC typically tests the analog voltage input to the device to digitally generated voltages, V_d, to decide which digitally generated voltage is closest. It usually uses a DAC to generate the V_d's. The digital number of the closest one is assigned to V_{in} by the device. Consider a 4 bit converter with a full scale voltage of 10V. The step size is 0.625V. If the V_{in}= 6.000V, closest to the digital voltage generated from $1010_b = 10_{10}$ which is 10×0.625V = 6.250V. Therefore it would be assigned the value 1010_b.

The digital number assigned to a voltage input, V_{in}, for a n-bit converter with a full scale voltage of V_{fs}, should be about

$$\frac{V_{in}}{V_{fs}} \times 2^n \qquad 3.$$

Of course it is rounded to the nearest whole number.

For instance a 9-bit converter with a full scale voltage of 10.24V and an input voltage of 3.600V should produce an output of $(3.600/10.24)\times 512 = 180_{10}$, but of course the output is typically in binary, not decimal, so it would be 0 1011 0100.

There are several ways of doing this.

1. **Staircase**. It compares the V_{in} to successive V_d's starting from the lowest and going toward the highest by increasing the digital number that generates V_d by one bit at a time. It stops when V_{in} becomes less than V_d and assigns that digital number to V_{in}. This can take some time if Vin is close to the full scale voltage. For an n bit converter it can take up to 2^n trials to do this. The timing of each comparison's is set by a clock, with one comparison each clock cycle. If the clock has a frequency of 1MHz it can take up to 256μs to do an 8 bit conversion. Staircase ADC's are not very fast and not used much any more.
2. **Successive Approximation**. This is a binary weighted testing scheme and it takes n comparisons to make an n bit conversion. Again the timing of each comparison is set by a clock. It would take 8μs to make an 8 bit conversion if the clock frequency is 1 MHz. These are fairly fast and are one of the most popular converter types.
3. **Flash.** Flash ADC's make all the comparisons simultaneously, essentially in one clock cycle. To do this requires 2^n comparators to compare V_{in} to 2^n V_d's. An 8 bit converter requires 256 comparators. A 10 bit one requires 1024. As a result, these tend to be expensive and are usually used where you need very fast conversions, e.g. a digital oscilloscope. Because of the exponential scaling of component number, they have usually been 8, 10 or 12 bit converters. However now there are 14 and even 16 bit versions. (These are often pseudo-flash converters.) Analog Devices sells a 2.6 billion sample per second, 2 channel, 14 bit converter for $1,100 (in 2018). Their 14bit 500 million samples per second converter is somewhat cheaper at $272.
4. **Dual Slope**. These are slow, but very accurate and fairly cheap. They, or their close relatives, are often used in Digital Multimeters. The "comparisons" are set by a clock again and it can take up to 2×2^n clock cycles per conversion.
5. **Delta Sigma**. The method here is somewhat complicated, but these can deliver high accuracy at surprisingly high speeds. There is one with a resolution of 32 bits at a sample rate of 4,000 samples/s. Audio recording systems often have ones that have 24 bits of resolution at either

96,000 or 192,000 samples/s. Since they are somewhat complicated, I won't talk about them, even though they have become quite popular in medium speed, high resolution systems.

The terms ***resolution***, ***accuracy***, ***step size*** and ***full scale voltage*** are also used here and have very similar meanings. Sometimes they talk about the voltage resolution of an ADC. This is the smallest change in voltage you can measure. It is basically the ***step size***. For an n bit converter it corresponds to $V_{fs}/2^n$. For an 8 bit converter with a $V_{fs} = 5.12V$, the voltage resolution is 20mV.

The ***dynamic range*** is the range of values the converter can span. For an n bit converter it is 2^n. For an 8 bit converter it is 256. Usually it is given in dB, or in this case $20 \text{ Log}_{10}(256) = 48\text{dB}$. For a 10 bit converter it is about 60dB. For a fair music system you would want a dynamic range of at least 80dB and many have a greater dynamic range.

Sampling

Often we want to digitize a waveform store it digitally and eventually replay it back as an analog signal, e.g. music (CD's) and digital oscilloscopes. One question that arises is how often do we need to sample the signal? When we use an ADC to measure a voltage, we have measured V(t) at a particular time. When we do this repeatedly, we have measured V(t) at a series of times. We do not have a continuous recording of the signal. This is sometimes called sampling. If we want a faithful reproduction of the original V(t), how often do we need to sample it? It depends on the range of frequencies in the signal. For instance in music, the range of human hearing is 20Hz to 20,000Hz. The range, or bandwidth, is 20,000Hz – 20Hz = 19,980Hz. The Nyquist theorem says that you need to sample at least twice the bandwidth, in this example it is 39,960 times per second, or roughly 40khz. (They often say 40kHz for 40,000 times per second even though Hz is really cycles per second, not times per second.) This rate is necessary to prevent aliasing and to get the correct amplitudes if you do a Fourier analysis of the signal. However to see something clearly on an oscilloscope, you usually need to sample 10 to 20 times the rate of the highest frequency in the signal. If I wanted to be able to accurately reproduce a 5kHz sine wave on the oscilloscope, I would probably want to sample at a rate of 100,000 times per second, or 100kHz.

A second consideration in sampling is that the input voltage should not change much in the time it takes to make a conversion, typically less than the step size or 1LSB. For a sine wave of amplitude A, the maximum rate of change of the signal is $A\times\omega$. If $A = 5V$ and $\omega = 10,000$, or $f = 1600Hz$, then the maximum rate of change is $50,000V/s = 50mV/\mu s$. For a 12bit converter with a full scale voltage of 10V, the step size is 2.4mV. In one microsecond the signal would change by almost 21 step sizes or 21 LSB! Not good. The solution is to use a **sample and hold amplifier** to capture the signal at an "instant" and hold that value while the converter makes the conversion. A reasonable sample and hold amplifier could "sample" the signal in a fraction of a microsecond. Then you have a record of what the voltage was at the time the sample and hold amplifier acquired the signal. Most good digitalization systems have a sample and hold amplifier to do this. (DMM's are an exception; they actually average the input signal over the measurement interval. They often average over a whole number periods for a 60Hz signal to eliminate 60Hz interference.)