Chapter 6: Active Filters

Op amps are the building blocks of more complicate systems and instruments. One important application is filtering. Often when you amplify a signal you also filter it to some extent. If the signals are clean to begin with, you don't need much filtering, but if they are small and noisy, you usually need some sort of filtering with the amplification. Filters made from op amps are usually called active filters, but many of them are really equivalent to an LCR circuit, i.e. they have the same transfer function, except they may also have a voltage gain as well.

A. RC Type Filters

The simplest filters are the standard high and low pass filter followed by a non-inverting op amp. In the case the amplifier primarily serves to buffer the filter.

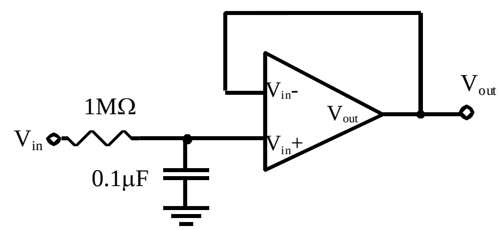

Fig. 6.1: Simple Filter

The filter shown is a low pass filter, but the output impedance of the RC stage is 1MΩ in parallel with 0.1µF. At low frequencies such that $\omega < 1/RC$, the output impedance approaches 1 MΩ. If you connect this directly to an oscilloscope whose input impedance is about 1 MΩ at low frequencies, the input impedance of the scope will change the characteristics of the filter. A JFET op amp following the RC stage will probably have an input resistance of > $10^{10}\Omega$. The output resistance of the op amp is very small so whatever follows the op amp will see a voltage source with an output resistance of < 1Ω.

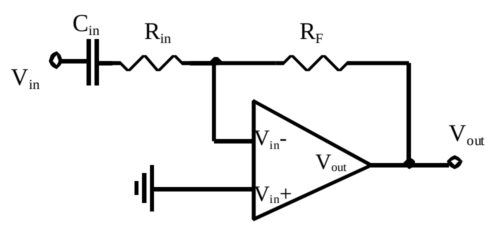

Fig. 6.2: High Pass Filter

A slightly more complicated filter is shown at the right. Here the input is a series RC circuit with a single resistor in the feedback loop between the output and the – input. The "gain" or the transfer function is just the ratio of the feedback impedance to the input impedance, or

$$H(j\omega) = -\frac{Z_F}{Z_{in}} = \frac{R_F}{R_{in} + 1/j\omega C} = -\left(\frac{R_F}{R_{in}}\right)\frac{j\omega R_{in} C}{1 + j\omega R_{in} C} \qquad 6.1$$

This is just – (R_F/R_{in}) times the transfer function for a high pass filter of cutoff frequency $\omega_c = 1/R_{in}C_{in} = 2\pi f_c$. If the capacitor was in parallel with R_F instead of in series with R_{in}, it would be a low pass filter with the same factor ,– (R_F/R_{in}), in front and a cutoff frequency of $\omega_c = 1/R_F C_F = 2\pi f_c$.. (I have called the capacitor C_F because in that case, it would be in the feedback loop instead of in the input.)

We can do both of these at the same time and the two filters will not interact. That is, we can have a series combination of R_{in} and C_{in} in the input and a parallel combination of R_F and C_F in the feedback loop. The transfer function is still $H(j\omega) = -\frac{Z_F}{Z_{in}}$.

This filter is shown at the right. You should try to work out the math to fill in the steps I've skipped in the equation below for the transfer function. Note that this is just the same as the two cascaded RC filters from chapter 3, except for the gain which is negative.

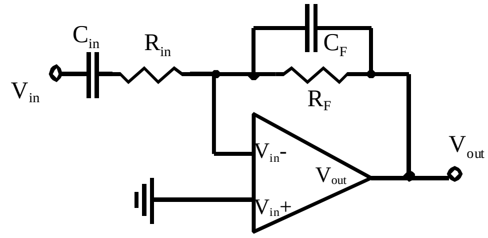

Fig. 6.3: Band Pass Filter

$$H(j\omega) = \left(-\frac{R_F}{R_{in}}\right)\frac{1}{1+j\omega R_F C_F}\left(\frac{j\omega R_{in} C_{in}}{1+j\omega R_{in} C_{in}}\right) \qquad 6.2$$

B. LCR Type Active Filters

The next examples are like LCR filters in that you can control the damping or Q of the circuit, which gives much more flexibility in designing filters. There are many ways of implementing these using one or more op amps. I will present a class of single op amp filters that have the same transfer function as an LCR circuit, but only use resistors and capacitors and may have an additional gain.

This type is called the equal component value Sallen-Key filter. (An alternative is the unity gain Sallen-Key filter, but I feel the equal component value is more flexible.) These filters use BOTH positive and negative feedback to produce the LCR effect. However, you have to make sure the negative feedback dominates or it will oscillate. In this circuit, $Z_1 = Z_3$ and $Z_2 = Z_4$, hence the "equal component value".

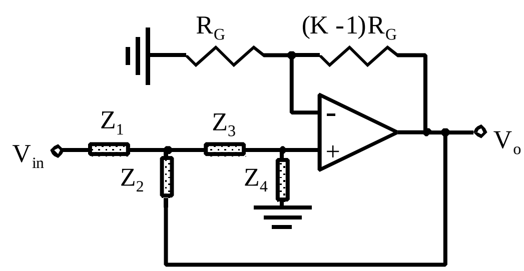

Fig. 6.4: Sallen-Key Equal Component Value Filter

If $Z_1 = Z_3 = R$, a resistance and $Z_2 = Z_4 = C$, a capacitance, it will be a second order low pass filter. The equivalent LCR undamped resonant frequency for this circuit is $\omega_o = 1/RC$. If $Z_1 = Z_3 = C$, a capacitance and $Z_2 = Z_4 = R$, a resistance, it will be a second order high pass filter with an equivalent LCR resonant frequency of $\omega_o = 1/RC$. The resistors R_G and $(K-1)R_G$ control the damping and the gain. The gain is K and the damping factor d = (3-K)/2. Note that if K =3, the damping is 0 and the circuit is on the borderline of oscillation. K > 3 will produce oscillations, i.e. it is no longer a linear amplifier.

As a design example, say we want a high pass filter with a damping factor of 0.6 and a "resonant" frequency of 300Hz. This would require that $Z_1 = Z_3 = C$ and $Z_2 = Z_4 = R$ where $1/RC = (2\pi)300$Hz, or $RC = 5.3\text{x}10^{-4}$s. A reasonable choice is R = 16kΩ and C = 33nF. This isn't exactly right, but those are 'standard' values for capacitors and resistors, and it is quite close to the desired value. (Standard off the shelf carbon resistors have a tolerance of 5% and the capacitors also have tolerances of 5 to 10%.) To get a damping factor of 0.6, one has 0.6 = (3 – K)/2 or K = 1.8. Then the feedback resistor is (K – 1) times $R_G = 0.8\ R_G$. 3.0kΩ and 2.4kΩ are also standard values and fit nicely. Note that the sum of the two is 5.4 kΩ so they won't require too much current from the op amp's output. The final circuit is shown at the right. Sometimes it is hard to find standard values of resistances and capacitances to match the frequency and damping. You can usually come close though. If not, you can choose a C and put two resistors in series to get the value you want, or use a variable resistor, i.e. a pot. If you want to be able to tune the filter, i.e. easily change ω_o or f_o, you can get a ganged pot. A ganged pot is actually two pots adjusted by the same shaft so that both pots will have about the same resistance. Thus you could simultaneously change both of the resistors that determine ω_o or f_o. These will have the same transfer function as an LCR circuit but multiplied by the gain K.

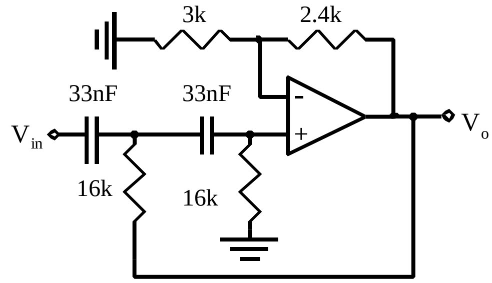

Fig. 6.5: Sallen-Key High Pass Filter

C. Band Pass Filters

You can also make active band pass filters with a single op amp, but they have more limitations than the high and low pass versions. You can get around these limitations if you go to multiple op amp versions like the state variable filter.

One example of a single op amp band pass is shown at the right. The resonant frequency is given by $\omega_o = 1/RC$ or $f_o = 1/(2\pi RC)$. Say you want a band pass filter that passes signals in the vicinity of 1600Hz with a Q of 5. (This means that the –3dB points are about 1440Hz and 1760Hz.) First you would note that f_o = 1600Hz means $\omega_o \approx 10{,}000$, and $RC = 10^{-4}$s. I want the input resistor R/2Q to be <u>at least</u> 1kΩ, so R = 1kΩx2Q = 10k. (For some reason, electronics data sheets and texts tend to drop the Ohms and write 5k for 5kΩ; probably a carryover from the distant past when it was harder to produce the Greek characters. For the same reason, sometimes you will find an m instead of μ, which is much more confusing.) If R=10k, 2QR = 100k. Since $RC = 10^{-4}$s and R = 10k, $C = 10^{-8}$F or 10nF.

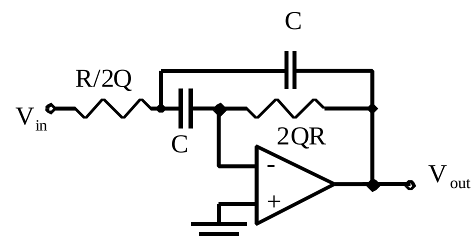

Fig. 6.6 Band Pass Filter

The gain of the circuit at resonance is $-2Q^2 = -50$. Therefore the open loop gain, A(ω), should be <u>at least</u> 5 times that, and preferably 10 times that at resonance. If the open loop gain needs to be 500 at 1600Hz, then the unity gain bandwidth will have to be 500×1600Hz = 800kHz. Most op amps will satisfy this. However, if you wanted to go to 50kHz with the same filter, the unity gain bandwidth would need to be 25MHz. You would need a fast op amp, or you would have to lower the Q. This is one of the problems with the single op amp version. Typically you are limited to low to moderate Q's or lower frequencies.

Probably the most versatile filter is the state variable filter. Its only drawback is that it requires three or four op amps. (You can get by with 3, but using 4 gives a little more flexibility.) This looks just like the LCR circuit, and you can control the resonant frequency, the gain and the Q of the circuit with independent adjustments. Also, the open loop gain requirements on the op amps are not so severe. There are commercial IC's that contain 4 op amps set up is this type of configuration. All you do is put in the resistors and maybe additional capacitors and your set to go. I've shown this filter below. The resonant frequency is $\omega_o = 1/RC$ and the damping is determined by R_d, the feedback resistor for amp 4. The damping will be

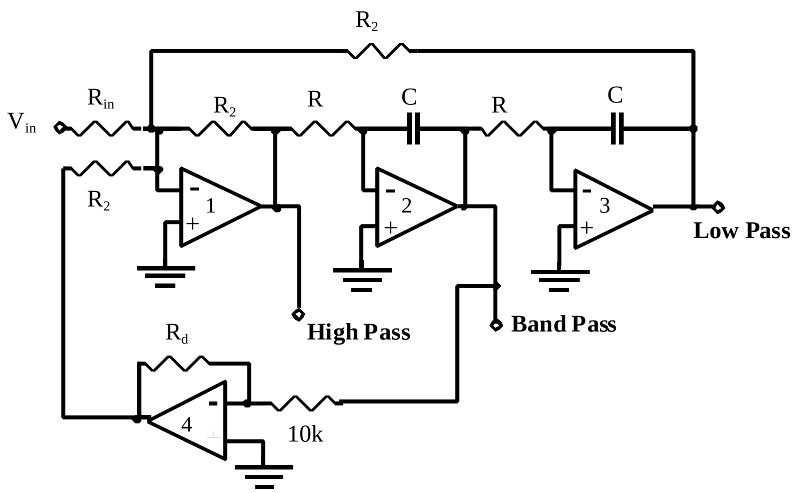

Fig. 6.7 State Variable Filter

given by $d = R_d/20k$, which is set by R_d and the 10k resistor in front of op amp 4. All three resistors labeled R_2 should be the same value, e.g. 10k. The overall gain is $-R_2/R_{in}$. You can replace op amp 4 by a resistor network if you want, but I prefer the 4 op amp version. Here you only need an open loop gain of 3Q or so at the resonant frequency for the band pass filter to be well behaved. Note that you can get any of the 3 outputs desired, high pass, low pass or band pass. If you sum the high and low pass with a 5th op amp, that output will be a notch filter at the resonant frequency.

There are many other filter types. A good book on active filters is <u>*Active Filter Cookbook*</u> by Donald Lancaster, but his damping factor is defined as $Q = 1/d$, not $Q = 1/2d$.