Chapter 5: Operational Amplifiers

This chapter will give an introduction to operational amplifiers, or op amps for short. They are one of the basic building blocks of analog circuits. They are used to amplify and perform certain mathematical operations on signals. They can also be used to generate signals, e.g as an oscillator.

A: BASICS

The symbol for an op amp is a triangle with two inputs and an output. There are at least two, and sometimes four, other connections that may or may not be shown, but are understood to be there. These other two connections are the power supply connections labeled V_{s+} and V_{s-}. You want to avoid applying a voltage to either input that does not lie between V_{s+} and V_{s-}, or you may damage it. Similarly, the output voltage cannot exceed V_{s+} or go below V_{s-}. The output voltage v_o is equal to a large number, A, times the difference of the voltage at two inputs v_+ and v_-, as long as $V_{s+} < v_o < V_{s-}$. If the difference between the inputs is too large and tries to drive it outside this range, the output is "pinned" at the maximum or minimum voltage it can achieve. (V_{s+} and V_{s-}. are often called the supply rails, so we say the output is pinned at one of the supply rails.) The large number A is called the open loop gain. Therefore

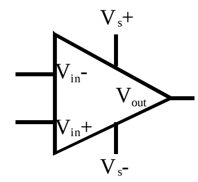

Fig 5.1

$$v_o = A(v_{in+} - v_{in-}) \tag{5.1}$$

Where A is typically between 10,000 and 1,000,000 (80 to 120dB). What do you do if you merely want to multiply an input signal by +10? The trick is negative feedback. Part of the output is "fed back" to the negative input, forming a feedback loop. (This is called closing the loop.) The circuit is shown at the right. You basically have three equations

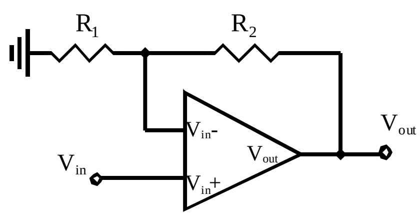

Fig. 5.2

1. $v_{in+} = V_{in}$ 5.2
2. $v_{out} = A(v_{in+} - v_{in-})$ 5.3
3. $v_{in-} = v_{out}\dfrac{R_1}{R_1 + R_2}$ 5.4

The last equation results from noting that R_1 and R_2 form a voltage divider from v_{out} to ground. I've also assumed that no current flows into or out of the v_{in-} input. You can use these three equations to find v_{out} in terms of V_{in}. Then

$$V_{out} = \frac{AV_{in}}{1 + \frac{AR_1}{R_1 + R_2}} = \frac{AV_{in}}{1 + A/G} \tag{5.5}$$

Where I've defined a new constant G by

$$G = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \tag{5.6}$$

One can solve eqn. 5.5 for the ratio $V_o/V_{in} = H$, which is real (in this approximation) and yields

$$H(j\omega) = \frac{A}{1 + A/G} \qquad 5.7$$

If A >> G then one has

$$V_{out} = HV_{in} \approx GV_{in} \qquad 5.8$$

For instance if G = 10 and A = 100,000, then eqn. 5.7 is accurate to 1 in 10,000 or 0.01%. This is why op amp manufacturers try to make A very large. Good op amps usually have A's close to 10^6, at least at low frequencies. In this approximation, the output is G times the input and G is the "voltage gain" of the circuit, or just the "gain". It is also called the closed loop gain because it is the gain of the circuit with the feedback loop completed or closed as opposed to the open loop gain A.

Therefore if I want to amplify a voltage by a factor of 10, I design a circuit like the one in fig. 5.2 and make $R_2 = 9R_1$. I would choose R_1 = 2k and R_2 = 18k. (I have chosen these because they are values you would find in a collection of 5% resistors.)

Most op amps do not like to supply more than 10 to 20mA of current at their output, so I try to keep the sum $R_2 + R_1$ between 5k and 100k if I can. (There are times to violate this, but only do so if you know what you're doing.)

Note that the effect of the negative feedback is to keep the two inputs, v_{in+} and v_{in-}, at the same potential to within a factor of G/A. Since G/A is very small, you can usually make the approximation that the two inputs are at the same potential, except when using eqn. 5.3. These lead us to the **GOLDEN RULES for op amps with negative feedback**.

1. **The inputs draw no current. (For FET op amps this is quite good in normal circuits since they usually draw < 1nA of current, and often < 0.1nA.)**
2. **The two inputs are at the same potential. This works as long as A >>G.**

This allows us to dispense with eqn. 5.3 in the above calculation and just use $v_{in+} = v_{in-} = V_{in}$ and

$$v_{in-} = V_{in} = v_{out}\,\frac{R_1}{R_1 + R_2} \qquad 5.9$$

This immediately gives us the gain as the output divided by the input or

$$G = 1 + \frac{R_2}{R_1} \qquad 5.10$$

This configuration is called a non-inverting amplifier, or a voltage follower with gain. (The non-inverting comes from the fact that the output has the same sign as the input. In the next example the output will have the opposite polarity relative to the input.)

The next example is an inverting amplifier. Here the gain G is negative. This circuit is shown at the right. **Note that there is still negative feedback!** Here the + input is grounded and instead the input signal goes to the – input through R_1, the lower side of the negative feedback loop. At first this looks formidable, but if you use the golden rules, it turns out

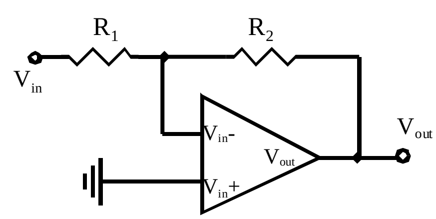

Fig. 5.3: Inverting Amplifier

to be easy. Rule 2 tells us that since $v_{in+} = 0$, so does v_{in-}. (As a result we say v_{in-} is a **virtual ground**, i.e. it is at 0V because the feedback forces it to match v_{in+}.) If that is true, then the current in R_1 can be calculated as

$$I_1 = V_{in}/R_1 \tag{5.11}$$

Now use rule 1 which says that no current flows into v_{in-}. This means that all this current through R_1 must continue through R_2 as I_2. The left side of R_2 is at 0V, so if the positive current flows from left to right,

$$V_{out} = - I_2R_2 = - I_1R_2 \tag{5.12}$$

Now use 5.11 to get

$$V_{out} = -V_{in}\frac{R_2}{R_1} = G\,V_{in} \tag{5.13}$$

Thus the gain of this amplifier configuration is

$$G = -\frac{R_2}{R_1} \tag{5.14}$$

This gain can approach 0, while the gain of the non-inverting amplifier is ≥ 1. The inverting amplifier presents an input resistance of R_1 to the voltage source, so you should choose R_1 >> than the output resistance of the source of V_{in}. The input resistance of the non-inverting amplifier is the input resistance of the op amp, which is usually large (see the discussion under D-1 below.)

These are the two basic configurations for op amps used as linear amplifiers. (There are other uses, but we'll come to them later.) It is worth reiterating that the output cannot go outside the range of the supply voltages. If you have supply voltages of +12V and –12V, the output cannot be greater than +12V or less than –12V, and the output usually can't quite get to the supply limits. (It varies with the type of op amp.) If your op amp's output is "stuck" at one of the supply voltages, you have probably connected something wrong.

Finally, even though we assume the inputs draw negligible current, they do draw some. If an input is not connected to anything, the input will usually drift toward one of the supply voltages.

EACH INPUT MUST HAVE A DC PATH TO GROUND THROUGH A RESISTOR OR A VOLTAGE SOURCE. DON'T LEAVE ONE UNCONNECTED!

B: MULTIPLE INPUTS TO AN OP AMP

In each of the cases above, there was only one signal input to the op amp. What happens if you have multiple inputs to an op amp? In fig. 5.4 V_1 is the input to v_{in+} and V_2 is the input to R_1 that connects to v_{in+} We use the golden rules to analyze this. We assume the two inputs, v_{in+} and v_{in-} are at the same potential, in this case V_1. Then the current through R_1 is given by

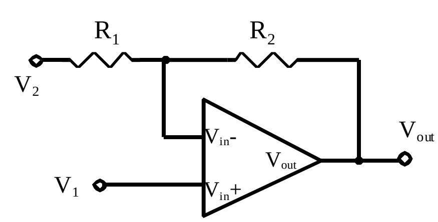

Fig. 5.4 Multiple inputs

$$I_1 = \frac{V_2 - V_1}{R_1} \tag{5.15}$$

since the drop across R_1 is $V_2 - v_{in-} = V_2 - V_1$. The output voltage is just $v_{in-} - I_1R_2$ since I_1 has to flow through R_2, there is nowhere else to go. This means that

$$V_{out} = V_1 - I_1R_2 = V_1 - \frac{V_2 - V_1}{R_1}R_2 = V_1\left(1 + \frac{R_2}{R_1}\right) - V_2\frac{R_2}{R_1} \qquad 5.16$$

The last expression consists of two parts. The first part containing V_1 is just the output we would get from a noninverting amplifier if V_2 was a ground. The second term involving V_2 is just what we would get from an inverting amplifier if V_1 was a ground.

The output voltage is the sum of what I would get if $V_1 = 0$, and V_2 isn't, plus what I would get if $V_2 = 0$ and V_1 isn't. The reason is that the system is linear, so the output is a linear combination of the two input voltages.

You should be able to apply this to fig. 5.5 where V_1 is connected to v_{in+} through a divider network, R_3 and R_4 to get the result in eqn. 5.17 below.

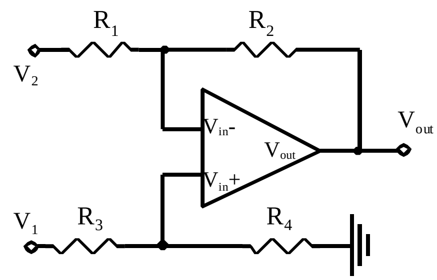

Fig. 5.5

$$V_{out} = V_1\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right) - V_2\frac{R_2}{R_1} \qquad 5.17$$

In the special case where $R_4/R_3 = R_2/R_1$, we have

$$V_{out} = (V_1 - V_2)\frac{R_2}{R_1} \qquad 5.18$$

This is called a difference amplifier, or a differential amplifier, since it amplifies the <u>difference</u> between the two input signals, V_1 and V_2. This configuration is used in instrumentation amplifiers. It is sometimes important to measure the difference in potential between two points, neither of which is grounded. You need a device that can accurately take the potential difference between two non-zero voltages in order to do that.

C: Summing Amplifier

Another useful configuration is shown at the right. It looks like an inverting amplifier, but it has two inputs. It is called a summing amplifier because the output voltage is the sum of the two inputs weighted by the two resistors R_1 and R_2. Again, the – input is a virtual ground so that the current flowing from V_{1in} and V_{2in} into the node where the three resistors meet is given by

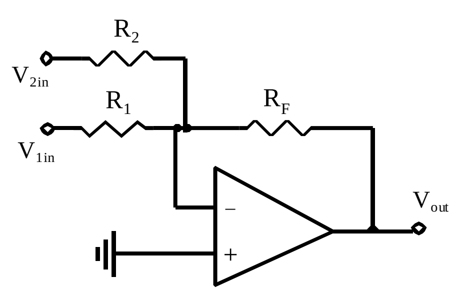

Fig. 5.6 A Summing Amplifier

$$I_{in} = \frac{V_{1in}}{R_1} + \frac{V_{2in}}{R_2} \qquad 5.19$$

All this current must flow through R_F so is the left side of R_F is at 0V, the right side is at

$$V_{Right} = V_{out} = -I_{in}R_F = -R_F\left(\frac{V_{1in}}{R_1} + \frac{V_{2in}}{R_2}\right) \tag{5.20}$$

One could add more resistors and inputs in parallel with R_1 and R_2 and they also sum to give

$$V_{out} = -R_F\left(\frac{V_{1in}}{R_1} + \frac{V_{2in}}{R_2} + \frac{V_{3in}}{R_3} + \ldots\right) \tag{5.21}$$

If all the R_{in}'s are the same = R, Vout is just $-R_F/R$ times the sum of the input voltages.

D: REAL OP AMPS

The golden rules apply to ideal op amps, but what about "real" op amps.

1. Input Currents.

The first thing to notice is that both inputs, v_+ and v_-, do draw some current. This current has two parts. The first is called a bias current, and it is approximately independent of the input voltage. It typically ranges from 100nA down to 1pA for different types of op amps. For FET input op amps, the bias currents are usually less than 1nA and often less than 100pA. (This varies with temperature for FET op amps, increasing with temperature.) Some specialty FET op amps have bias currents < 1pA. The second part of the input current varies with input voltage and is referred to as an input resistance or impedance. Input impedances range from about $10^7\Omega$ to > $10^{12}\Omega$. The larger input impedances are usually associated with FET op amps. The large input impedance and low bias current are why I usually try to use FET op amps unless I have a specific reason for not doing so. The input bias currents and input resistance are more of a problem when the voltage source driving the op amp has a large output resistance, i.e. Thevenin resistance. This becomes apparent if you try to measure the voltage across a 0.1μF capacitor. A bias current of 10nA will make the voltage across the capacitor change by 100mV every second! A bias current of 10pA will make it change by 0.1mV every second.

2. Open Loop Gain.

I mentioned that the open loop gain A is typically $10^4 < A < 10^6$. As long as G << A, the two inputs will be at approximately the same potential when there is negative feedback. However, the open loop gain is actually a function of frequency. It typically has the form

$$A(j\omega) = \frac{A_o}{1 + j\omega\tau_o} = \frac{A_o}{1 + j\left(\frac{f}{f_o}\right)} \tag{5.22}$$

This looks like a low pass filter. The cutoff frequency f_o is usually between 10 and 100Hz. $|A(j\omega)|$ will reach 1 when $A_o f_o/f = 1$, or $f = A_o f_o$. If $A_o = 10^6$ and $f_o = 10$Hz, this will occur at $f_{UG} = 10^7$Hz. That frequency is called the unity gain bandwidth, which is why I've labeled it f_{UG}. Typical unity gain frequencies or bandwidths are between 1MHz and 50MHz. (Specialty products can have higher bandwidths than 50MHz and low power devices often have bandwidths less than 1MHz.)

At frequencies above f_o, $A(j\omega) \approx A_o f_o/(jf)$. If you insert this into eqn. 5.5 for A, and assume that $A_o >> G$, where G is the DC gain given by eqn. 5.6 or 5.14, then you will see that, after some algebra

$$V_{out} = GV_{in} \frac{1}{1 + j\left(\frac{fG}{A_o f_o}\right)} = GV_{in} \frac{1}{1 + j\frac{f}{f_e}} \qquad 5.23$$

where $f_e = A_o f_o/G$ = the effective cutoff frequency = f_{UG}/G. The output is G times the input times what looks like a low pass filter with a cutoff frequency of f_e, resulting in a transfer function of

$$H = G\frac{1}{1 + j\left(f/f_e\right)} \qquad 5.24$$

As long as $f << f_e$, the normal gain equation works, but as f approaches f_e, the gain will decrease, just like the transfer function for a low pass filter. By the time $f = f_e = f_{UG}/G$, the magnitude of the transfer function is no longer G, but 3dB below G or 0.707G. Therefore if you have an op amp with a bandwidth of 10MHz and want a gain of 10, the magnitude of the transfer function will drop to 7 at 10MHz/G or 1MHz. At 140kHz it would be about 9.9.

If you want an amplifier with a gain of 50 at 20kHz, e.g. for audio purposes, you will probably want a unity gain frequency of close to 10MHz so that f_e = 200kHz. This means f_e is well above the frequency of interest and that the op amp will behave almost ideally at the frequency of interest.

3. Slew Rate

The slew rate is the maximum value of (dV/dt) at the output. It is usually written as volts per microsecond, or V/μs. A typical or general purpose op amp would have a slew rate of 1 to 20 V/μs. If you had an op amp of closed loop gain 10 whose slew rate was 10V/ μs and the input went from 0.0V to 0.5V in 0.01 μs, the output would change toward + 5V, but it would take about 0.5μs for it to reach 5V because it can only go a maximum of 10V every microsecond.

4. Offset Voltage

If you have arranged your amplifier to be a non-inverting amplifies with a gain of 10 and ground the input, you would expect the output voltage to be 0.000V. However, it might actually be 20mV, or anywhere between ±20mV. This is because the two inputs are not exactly matched. This mismatch produces an offset voltage. In this case it is 20mV/10 = 2mV. (It gets amplified by the gain.) A gain of 100 and a 0.000V input would produce a 200mV output. This offset can be ±. Most op amps have a means of nulling this with a pot. However this offset changes as the temperature of the amplifier changes. For typical op amps this can change by 5 to 20μV per deg. C. For most applications, this is not a problem, but when you are trying to measure tens or even hundreds of μV, this drift can be a nuisance. (Special op amps and instrumentation amplifiers can have very small offsets and drifts, often more than a factor of 10 better than general purpose op amps.)

5. Power Supplies

Most bipolar and JFET op amps are designed to operate with a difference between V_{S+} and V_{S-} of 10 to 36 volts. They may not work well if the difference is 8V and they may “die” if it exceeds 36V for an extended period. Most people use values for V_{S+} and V_{S-} of ±12V to ±15V for bipolar and JFET op amps. The output voltages cannot exceed V_{S+} or go below V_{S-}. If the supply voltages are ±12V, many op amps won’t go past ±10V at the outputs. Most CMOS op amps want the difference between V_{S+} and V_{S-} to be < 18V. However, their output may go all the way from V_{s+} and V_{s-}. (The terminology is rail to rail. V_{s+} is often called the positive rail and V_{s-} is called the negative rail.)

Warning!

The quickest way to destroy an op amp is to accidentally connect the power supplies up backwards. The second quickest way is to connect the output to one of the supply voltages. (The outputs are usually protected against accidental grounding, but do not always survive being connected to V_{s+} or V_{s-}.)

6. Stability

When you produce a negative feedback loop around an op amp, it should be stable, i.e. not oscillate. A small amount of positive noise, δ, at the output of the amplifier in fig. 5.2 would feed back to v_{in-}, but at the negative input it would have the effect of causing the output to decrease back toward the value of the output before the noise occurred, which I'll call the equilibrium value. Similarly a noise of – δ at the output would cause the output to increase back toward the previous equilibrium value. The response of the system to a disturbance, i.e. noise, is to return to the previous equilibrium value. It is a stable equilibrium, like a ball bearing in the bottom of a bowl. If you connected the feedback loop to the + input, it would be just the opposite, it would be an unstable equilibrium and the first bit of noise would sent the output to one of the supply rails, or a close as it could get to one.

There is one final aspect to stability and that concerns phase shifts in the signal. Even with resistors in the feedback loop, the transfer function still has a phase shift at higher frequencies. If that phase shift were enough, the signal coming back to the – input would look like positive feedback and the amplifier could oscillate. Most op amps are internally "compensated" so that a signal in a negative feedback loop made of resistors will not have enough phase shift to become unstable. (This compensation artificially slows down the op amp, but that is usually a small price to pay for stability.) However, if you put capacitors and inductors in the feedback loop, it is possible to introduce enough extra phase shift to make it act like positive feedback and therefore be unstable.

I have discussed general purpose op amps here, but I have referred to other types as well. There are lots of op amps designed for special purposes. Some are designed for operation at higher frequencies. Others are designed to work with batteries as their power supplies and they typically do not draw much current from their power supplies if they don't have to put out much current to drive the load they are connected to. Some are designed to operate with low voltage power supplies, even a single 1.6V supply. Some are designed for precision measurements that require a large open loop gain and low offset voltages. These are only a few of the special types you could encounter. It is hard to make one op amp that will satisfy all of these requirements, that is why there are so many types of op amps. The general purpose op amps attempt to satisfy all these to <u>some</u> extent, and still be reasonably inexpensive. We will use a general purpose op amp in our laboratories.