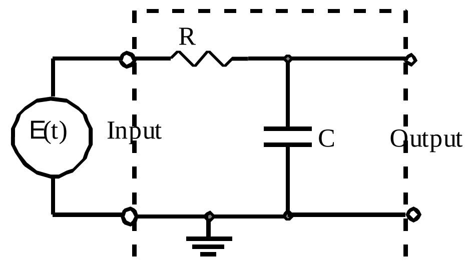
# Chapter 3:  Passive Filters and Transfer Functions

In this chapter we will look at the behavior of certain circuits by examining their transfer functions. One important class of circuits is filters. A good example is trying to tune in a radio station. If you want to listen to an FM station broadcasting at 89.3Mhz, you want to process the signal coming from your antenna and allow any signal very close to 89.3MHz to pass on while blocking, or attenuating, all other signals. Circuits that allow voltages at some frequencies to pass while attenuating those at other frequencies are called filters.

## A: FIRST ORDER LOW PASS FILTERS

The simplest filters, and crudest, are first order high pass filters and first order low pass filters. These can be made of a resistor and a capacitor or made of a resistor and an inductor. (Resistors and capacitors are usually used at low frequencies.)We looked at one in the previous chapter and it is shown in fig. 2.3 which I've duplicated at the right. This is often called a low pass RC filter. We are primarily interested in the complex transfer function, H(jω) and particularly




in its polar representation, |H(jω)| exp(jϕ). The magnitude of H(jω), or its amplitude, is (from eqn. 2.40)

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(\omega\tau)^2}} \qquad 3.1$$

where I've used $\tau = RC$. The phase of H(jω), ϕ, is just

$$\phi = -\tan^{-1}(\omega\tau) \qquad 3.2$$

We usually plot these as a function of ω for a given τ, but first let's look at the general behavior of these functions. (Note that all circuits with the same product of R and C yield the same curve or behavior.)

Case 1: If $\omega\tau << 1$, the magnitude of $H(j\omega) \approx 1$ and $\phi \approx 0$. In this case the output is approximately the same as the input, i.e. the same amplitude and little phase shift.

Case 2: If $\omega\tau >> 1$, the magnitude of $H(j\omega) \approx 1/\omega\tau << 1$ and the phase shift $\phi \approx -\pi/2$ or $-90^o$.

Case 3: If $\omega\tau = 1$, the magnitude of $H(j\omega) = 1/\sqrt{2} = 0.707$ and the phase shift $\phi = -\pi/4$ or $-45^o$.

This suggests that the circuit attenuates and shifts the phase of signals whose frequency $\omega > 1/\tau$. $1/\tau$ is usually called the cutoff frequency, $\omega_c$ for the circuit, since that is where the behavior changes, even though the change is gradual for this type of circuit. (The cutoff frequency is usually **defined** as the frequency where the output amplitude is – 3dB from the input amplitude.) One could guess this result by noting that the impedance of the capacitor decreases as the frequency increases. At low frequency, the capacitors impedance is large and almost all the voltage drop is across the capacitor. At $\omega\tau = 1$ the magnitude of the capacitors impedance equals the resistors, and the voltage is split between them. (Note that each can have 70.7% of the voltage drop instead of 50% because the maximum voltages drops are out of phase with each other by π/4 at that point, i.e. they do not occur at the **SAME TIME**. At the same time, the sum of the two drops must equal the input voltage.) At high frequencies, the impedance of the capacitor is small and almost all the voltage drop is across the resistor.

Since we are often interested in the behavior over a wide range of frequencies and attenuations, we use a logarithmic scale. We plot $20Log_{10}(|H(j\omega)|)$ versus the $Log_{10}(\omega/\omega_c)$. Note that H is dimensionless, so it is ok to take the log of H. (I've dropped the [jω]. Also, if I use Log or log that will be base 10 and ln will be the natural logarithm, or base e.) When we use $20Log_{10}(|H(j\omega)|)$, this is $|H(j\omega)|$ in dB or decibels. The table at the right shows several useful relations for dB. Note that if $|H| < 1$ it's log is negative. The H at the right is not one for any particular circuit; I've just made up some values for an example.

| $\lvert H(j\omega)\rvert$ | $\lvert H(j\omega)\rvert$ in dB |
|---|---|
| 10 | 20dB |
| 1 | 0dB |
| 0.707 | −3dB |
| 0.5 | −6dB |
| 0.1 | − 20dB |
| 0.01 | − 40dB |

Note that |H| is the ratio of the magnitudes, or amplitudes, of the output voltage to the input voltage. Thus the dB scale here uses the input amplitude as a reference and measures the output amplitude as a fraction of the input's amplitude. (We also use a dB scale for other quantities like sound intensity, where the reference sound intensity is $10^{-12}W/m^2$.) For the **low pass filter** example, if $\omega\tau = 1$, the magnitude of H = − 3dB, or the output is 3dB **below** the input level or 0.707 times the input level. The two plots below show |H| as a function of $f = \omega/2\pi$ with the |H| in dB and f as $Log_{10}(f/f_c)$. (Note that $f/f_c = \omega/\omega_c$.) The second plot shows the phase vs. $Log_{10}(f/f_c)$. We usually plot these vs $Log_{10}(f/f_c)$ because they scale as $(f/f_c)$. **Every first order low pass filter will have the same shape when plotted this way.**

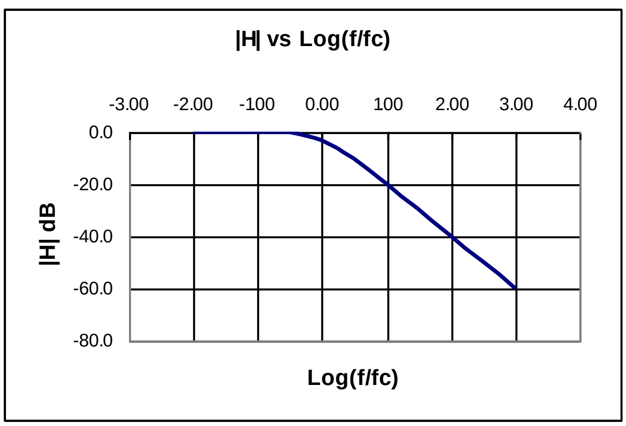



Fig. 3.1

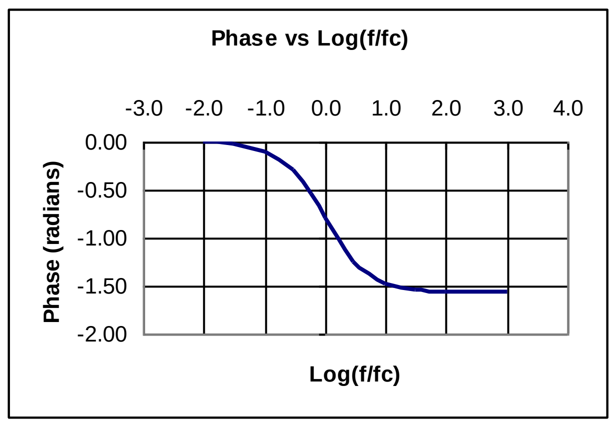



Fig. 3.2

The filter is called a first order filter because in the region where there is attenuation the output amplitude is approximately proportional to $(1/\omega)$ to the first power. A second order low pass filter would be approximately proportional to $(1/\omega)^2$ in that region. (How would a 3rd order low pass filter depend on $(1/\omega)$ when $\omega >> \omega_c$?)

Try to answer the following 3 questions from the first graph.

a) If $f_c$ = 1000Hz, where will the output amplitude be 1/10 the input amplitude?

b) If $f_c$ = 10Hz, where will the output amplitude be 1/100 of the input amplitude?

c) If I want the output amplitude at 60 Hz to be 1/10 of the input amplitude, what should the cutoff frequency $f_c$ be?

If $\omega\tau >> 1$, or $\omega >> \omega_c$, $H(j\omega) \approx 1/(j\omega\tau)$, or the complex $V_{out}$ equals the integral of $\mathcal{E}_{in}$ times $1/\tau$. (You should verify this statement.) **This circuit approximately integrates the input voltage if $\omega\tau >> 1$.**

## B: FIRST ORDER HIGH PASS FILTERS

A first order high pass filter will be similar to the low pass filter, but the capacitor and resistor will be interchanged, i.e. the output voltage will be the voltage across the resistor. The circuit is shown at the right. Again the input is a sinusoidal voltage and we will use its complex representation. This circuit is just a divider circuit, but with the impedances $Z_C$ and $Z_R$ reversed in position from the low pass example. The complex output voltage will be given by

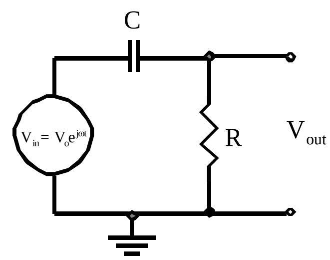



fig 3.3 High Pass Filter

$$V_{out} = V_{in}\frac{Z_R}{Z_R + Z_C} = V_{in}\frac{R}{R + \frac{1}{j\omega C}} = V_{in}\frac{j\omega RC}{1 + j\omega RC} \qquad 3.3$$

The transfer function $H(j\omega)$ is just the coefficient of $V_{in}$ or, using $RC = \tau$,

$$H(j\omega) = \frac{j\omega\tau}{1 + j\omega\tau} = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}\exp\left(j\left\{\frac{\pi}{2} - \tan^{-1}(\omega\tau)\right\}\right) \qquad 3.4$$

The last term has been converted into polar coordinates. (You should verify that this is the correct form.) The magnitude and phase of H are given by

$$|H(j\omega)| = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}} \quad \text{and} \quad \varphi = \text{phase} = \frac{\pi}{2} - \tan^{-1}(\omega\tau) \qquad 3.5$$

Again we will consider three cases:

Case 1: If $\omega\tau << 1$, $|H(j\omega)| \approx \omega\tau$ and $\phi \approx \pi/2$ or $90^o$. The output is smaller than the input, and the phase shift approaches $90^o$.

Case 2: If $\omega\tau >> 1$, $|H(j\omega)| \approx 1$ and the phase shift $\phi \approx 0$.

Case 3: If $\omega\tau = 1$, $|H(j\omega)| = 1/\sqrt{2} = 0.707$ and the phase shift $\phi = \pi/4$ or $45^o$.

This is called a high pass filter because frequencies above $\omega_c = 1/\tau$ tend to be passed with little attenuation or phase shift while those below $\omega_c$ tend to be attenuated. Notice the phase shift here is positive while for the low pass it was negative. The magnitude of H (in dB) and the phase of H (in radians) are plotted below versus $\text{Log}(f/f_c)$. It is called a $1^{st}$ order filter because |H| goes

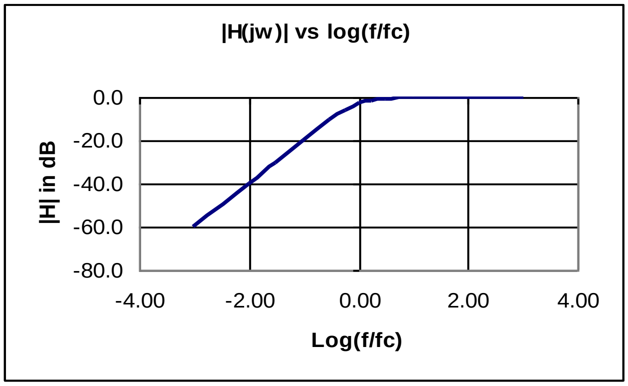



Fig. 3.4

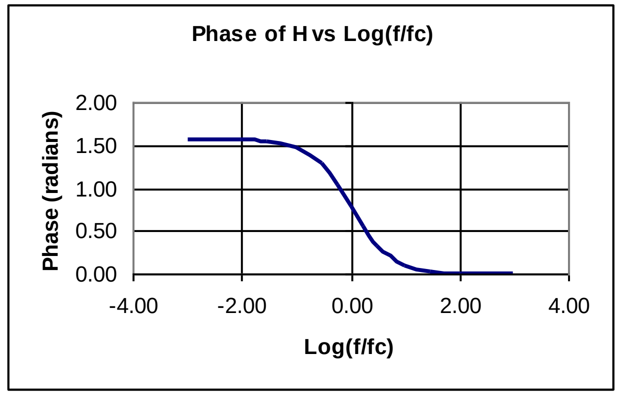



Fig. 3.5

like $(f/f_c)$ to the first power when $f << f_c$. (Can you guess how a 2nd order high pass filter's transfer function would behave when $f << f_c$?) **<u>All 1st order high pass filters have the same shape when plotted this way</u>.** The transition from the region of little attenuation, $f >> f_c$, to the region of strong attenuation is not very sharp with this type of filter, the transition region being roughly from $(f/f_c) = 1/3$ to $(f/f_c) = 3$. That is the $|H_{HP}| = 0.95$ at $(f/f_c) = 3$, within 5% of the high frequency limit and $|H_{HP}| = 0.316$ at $(f/f_c) = 1/3$, where the approximate expression yields 0.333, again a deviation of about 5%. For many applications we can approximate $|H_{HP}| = 1$ if $(f/f_c) > 3$ and $|H_{HP}| = (f/f_c)$ if $(f/f_c) < 1/3$.

The first order high pass filter blocks the DC or constant part of a signal, and only passes the part that depends on time. For example, if the input is 5V + Acos($\omega$t) and $\omega >> \omega_c$, the output will be just Acos($\omega$t). The 5V will be blocked and disappear. The inputs to some devices, e.g. oscilloscopes, have a choice or **<u>AC or DC coupling</u>**. DC coupling passes all parts of a signal. AC coupling puts the input through a high pass filter, which blocks the lower frequencies. The RC time constant for an oscilloscope is usually around 0.1s, producing a cutoff frequency of about 1.6 Hz. When a signal goes through a high pass filter, it is shifted so that for times $>> \tau$, the <u>average</u> of the output voltage is 0 volts.

At frequencies below the cutoff frequency, this circuit approximately differentiates the input and multiplies it by $\tau$ or $1/\omega_c$, i.e. $|H(j\omega)| \approx \omega\tau$ or $\omega/\omega_c$. (You should verify this.)

Finally, it is often helpful to write the transfer functions of these filters in terms of the cutoff frequency $\omega_c = 2\pi f_c = 1/\tau$. Then the first order low pass filter has a transfer function given by

$$|H_{LP}| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}, \qquad \text{and} \qquad \phi_{LP} = -\tan^{-1}\left(\frac{f}{f_c}\right) \qquad 3.6$$

and the first order high pass filter has a transfer function given by

$$|H_{HP}| = \frac{f/f_c}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}, \qquad \text{and} \qquad \phi_{HP} = \frac{\pi}{2} - \tan^{-1}\left(\frac{f}{f_c}\right) \qquad 3.7$$

**If one defines $x = f/f_c = \omega/\omega_c$, they take on a somewhat neater form.**

$$|H_{LP}| = \frac{1}{\sqrt{1+x^2}}, \qquad \text{and} \qquad \phi_{LP} = -\tan^{-1}(x) \qquad 3.8$$

$$|H_{HP}| = \frac{x}{\sqrt{1+x^2}}, \qquad \text{and} \qquad \phi_{HP} = \frac{\pi}{2} - \tan^{-1}(x) \qquad 3.9$$

**A useful approximation is that for x >> 1,**

$$|H_{LP}| \approx \frac{1}{x} \text{ and } |H_{HP}| \approx 1$$

**Similarly, for x <<1**

$$|H_{LP}| \approx 1 \textit{ and } |H_{HP}| \approx x$$

## C: SECOND ORDER RC-LIKE FILTERS

The simplest second order filter is two first order filters that are cascaded, i.e. one follows the other in the circuit. However, for it to behave nicely, the stages should be non-interacting. This will be approximately the case when the Thevenin impedance of the first stage (its output impedance) is much larger than the series impedance of the two elements of the second stage. Note that in the circuit in fig. 3.6, a second order low pass filter, I've chosen $R_2 = 10R_1$ and $C_2 = C_1/10$ which ensures that the series impedance of the second stage is at least 10 times the output impedance of the first stage. That is usually sufficient. If that is the case, the transfer function for the entire circuit is, to a good approximation, the product of the two separate transfer functions, or $H_{total}(j\omega) \approx H_1(j\omega) \times H_2(j\omega)$. This makes it much easier to analyze the circuit. In this example both circuits "accidentally" have the same RC time constant $\tau$. (They don't have to have the same $\tau$, but I'll leave some of those cases for you to figure out.) As a result,

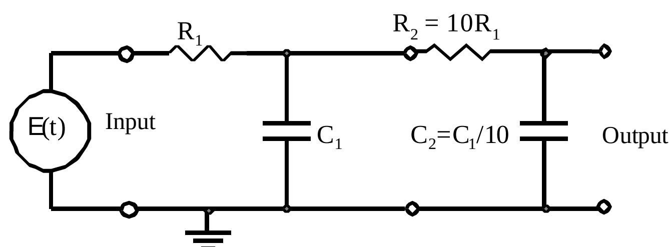



Fig. 3.6

$$H_{total}(j\omega) = \left(\frac{1}{1+j\omega\tau}\right)^2 = \frac{1}{1+(\omega\tau)^2}\exp\left(-2j\tan^{-1}(\omega\tau)\right) \qquad 3.10$$

where the last expression is in polar form. Again we will look at three cases.

Case 1: If $\omega\tau << 1$, the magnitude of $H(j\omega) \approx 1$ and $\phi \approx 0$. In this case the output is approximately the same as the input, i.e. the same amplitude and no phase shift.

Case 2: If $\omega\tau >> 1$, $|H(j\omega)| \approx (1/\omega\tau)^2 << 1$ and the phase shift $\phi \approx -\pi$ or $-180^o$.

Case 3: If $\omega\tau = 1$, the magnitude of $H(j\omega) = 1/2$ and the phase shift $\phi = -\pi/2$ or $-90^o$.

The reason it is called a second order filter is that the amplitude falls off like ($1/\omega\tau$) to the second power if $\omega\tau >> 1$. You should also note that the amplitude of $H_{total}$ is the product of the two individual amplitudes and the phase is the sum of the two individual phases. At high frequencies, the transfer function ≈ $(1/\omega\tau)^2$. In this region a doubling of the frequency results in a reduction of 4 in the amplitude, or the amplitude changes by −12dB for every doubling of the frequency. (In audio systems they usually say −12dB per octave.)

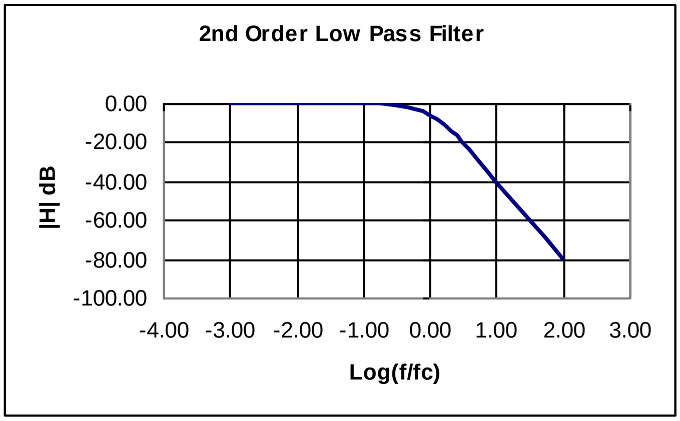



Fig. 3.7

Cascading two non-interacting filters means that the net transfer function is the product of the two individual transfer functions. When I use the dB or logarithmic scale,

$$20 \text{ Log}(|H_{net}|) = 20 \text{ Log}(|H_1|\times|H_2|) = 20 \text{ Log}(|H_1|) + 20 \text{ Log}(|H_2|),$$

or in dB, $|H_{net}| = |H_1| + |H_2|$. **In the dB scale they add.** This is a useful shortcut when plotting the $|H_{net}|$ on a dB scale. It is interesting to note that the phase shifts also add.

This is not a very sharp filter, i.e. the transition from $|H| \approx 1$ to the region above $\omega\tau = 1$ where it is decreasing rapidly occurs over a wide range of frequencies. For instance if you want to pass a signal of frequency $f_s$ and not attenuate it by more than 10%, i.e. $|H| = 0.90$, you need to set $f_c$ of the individual filters such that

$$0.90 = \frac{1}{1+\left(f_s/f_c\right)^2} \qquad 3.11$$

or $(f_s/f_c) = 1/3$. This means that $f_c = 3f_s$, the cutoff frequency of each filter stage has to be 3 times the signal frequency. A noise signal around $f_c$ would only be reduced in amplitude to ½ its original amplitude. The noise signal would have to be at $3f_c = 9f_s$ to be reduced in amplitude by a factor of 10 (−20dB.)

There are three different versions of cascading two first order RC type filters to get a second order filter. The first is cascading two low pass filters. The second is cascading two high pass filters and the third is the cascade a low pass and a high pass filter to produce a band pass filter. The band pass filter would like the one below, where I've again made the two cutoff frequencies the same. I've also show a plot of |H|. Note that |H| has a maximum of – 6dB or 1/2. (Often you want the low pass cutoff frequency higher than the high pass cutoff frequency.) I will leave these for you to investigate.

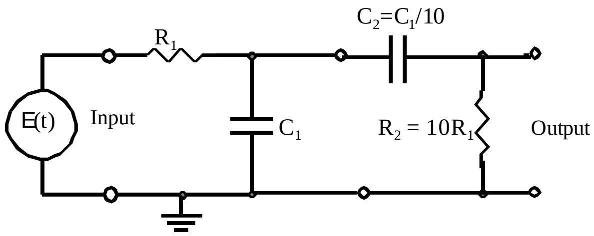



Fig. 3.8

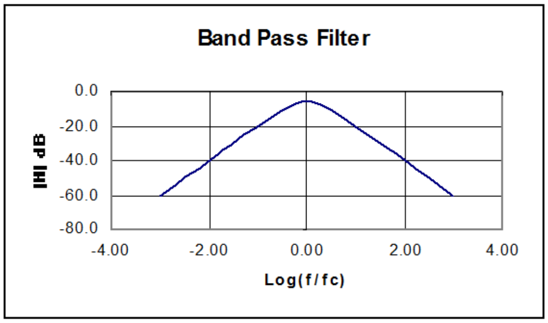



Fig. 3.9

## D: SECOND ORDER LCR FILTERS

A "better" second order filter can be made from an inductor, capacitor and a resistor in series. You can get all three behaviors by looking at the voltage drop across different elements. If your output is the voltage across the capacitor, you will have a second order low pass filter. If the output is across the inductor, you will have a second order high pass filter. If the output is across the resistor, you will have a band pass filter. Consider the low pass filter version. It is shown at the right. This looks just like another divider circuit. If the input voltage is $V_o\exp(j\omega t)$, the output voltage $V_{out}$ will be

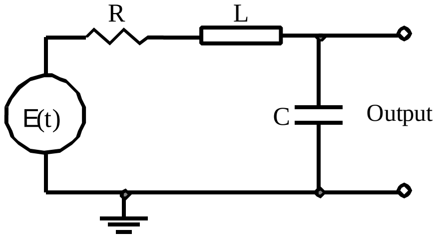



Fig. 3.10

$$V_{out} = \frac{Z_C}{Z_C + Z_L + Z_R} V_o \exp(j\omega t) \qquad 3.12$$

and the transfer function $H_{LP}(j\omega)$ will be given by $V_{out} = H\,V_{in}$. Therefore

$$H_{LP}(j\omega) = \frac{Z_C}{Z_C + Z_L + Z_R} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L + R} = \frac{1}{1 - LC\omega^2 + j\omega RC} \qquad 3.13$$

Normally one sets $LC = (1/\omega_o)^2$ since LC has units of 1 over frequency squared and this $\omega_o$ is the angular frequency of free oscillations for an LC circuit, sometimes called the natural frequency or undamped resonant frequency for the circuit. One can write

$$H_{LP}(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\omega RC} = \frac{\omega_o^2}{\omega_o^2 - \omega^2 + j\omega\omega_o^2 RC} \qquad 3.14$$

The magnitude of H is just

$$\left|H_{LP}(j\omega)\right| = \frac{1}{\sqrt{\left(1 - \left(\omega^2/\omega_o^2\right)\right)^2 + \omega^2 R^2 C^2}} = \frac{\omega_o^2}{\sqrt{\left(\omega_o^2 - \omega^2\right)^2 + \omega^2 \left(R/L\right)^2}} \qquad 3.15$$

The phase of H, $\phi$, is

$$\varphi_{LP} = -\tan^{-1}\left(\frac{\omega R/L}{\omega_o^2 - \omega^2}\right) \qquad 3.16$$

Again we will consider three cases:

Case 1: If $\omega \ll \omega_o$, the magnitude of $H_{LP}(j\omega) \approx 1$ and $\phi_{LP} \approx 0$. In this case the output is equal to the input.

Case 2: If $\omega \gg \omega_o$, the magnitude of $H_{LP}(j\omega) \approx (\omega_o/\omega)^2$ and the phase shift $\phi_{LP} \approx -\pi$ or $-180^o$. The output's magnitude is much less than the input's.

Case 3: If $\omega = \omega_o$, the phase shift $\phi_{LP} = -\pi/2$ or $-90^o$ and the magnitude of $H(j\omega)$ is

$$|H_{LP}(j\omega_o)| = \frac{1}{\omega_o RC} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad 3.17$$

where I've put $\omega = \omega_o = 1/(LC)^{1/2}$ in eqn. 3.11. The interesting thing is that the output can be larger than the input for certain choices of R, L and C. For instance if L = 1mH, C= 1nF, then $(L/C)^{1/2} = 1000$. If $R < 1k\Omega$, the output is larger than the input at this frequency. There are two terms associated with this circuit, the Q of the circuit and the damping factor, d. Most people define d = 1/(2Q), but some define d as d = 1/Q. (It is d that changes from author to author, not the Q.) **I'll use d = 1/(2Q)**.

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2d} \qquad 3.18$$

Q and d are dimensionless. **At $\omega = \omega_o$, |H| = Q.** If $d = 1/\sqrt{2} = 0.707$, the output amplitude is never larger than the input's. If d < 0.707, then for some frequency, the output amplitude will be larger than the input's. If d =1, the circuit is said to be critically damped, and if d >1, the circuit is said to be overdamped. These terms are also used to describe mechanical oscillations where the symbol for the damping factor is usually $\gamma$. If one lets $x = (\omega/\omega_o)$, one can rewrite |H| as

$$|H_{LP}(j\omega)| = \frac{1}{\sqrt{(1-x^2)^2 + 4x^2d^2}} \qquad 3.19$$

Usually 0< d < 1, so for x <<1 and x>>1 the magnitude of H is not sensitive to d. It is in the region around x=1 that the value of d is important, typically for (1/3) < x < 3. (This applies to the high and low pass outputs, not to the band pass output.) You should also note that the maximum value of |H| is not necessarily at x = 1, although it goes to x = 1 as d gets smaller. The maximum of H occurs when the denominator in 3.15 is a minimum. If you take the denominators derivative and set it = 0, you'll find that the maximum for |H| occurs at $x^2 = (1 - 2d^2)$ if $d < 1/\sqrt{2}$. If $d > 1/\sqrt{2}$, the maximum is at x =0, or $\omega = 0$. {Note that I can substitute $f/f_o$ = for $\omega/\omega_o$ in 3.15 above. This is because the $2\pi$'s cancel. It is often convenient to use $x=f/f_o$ instead of $x=\omega/\omega_o$ when doing calculations.) I have plotted |H| in the region 0.1 < x < 5 for several d's below. T**he d=1 case is the same as two cascaded RC filters with the same $\tau$.**

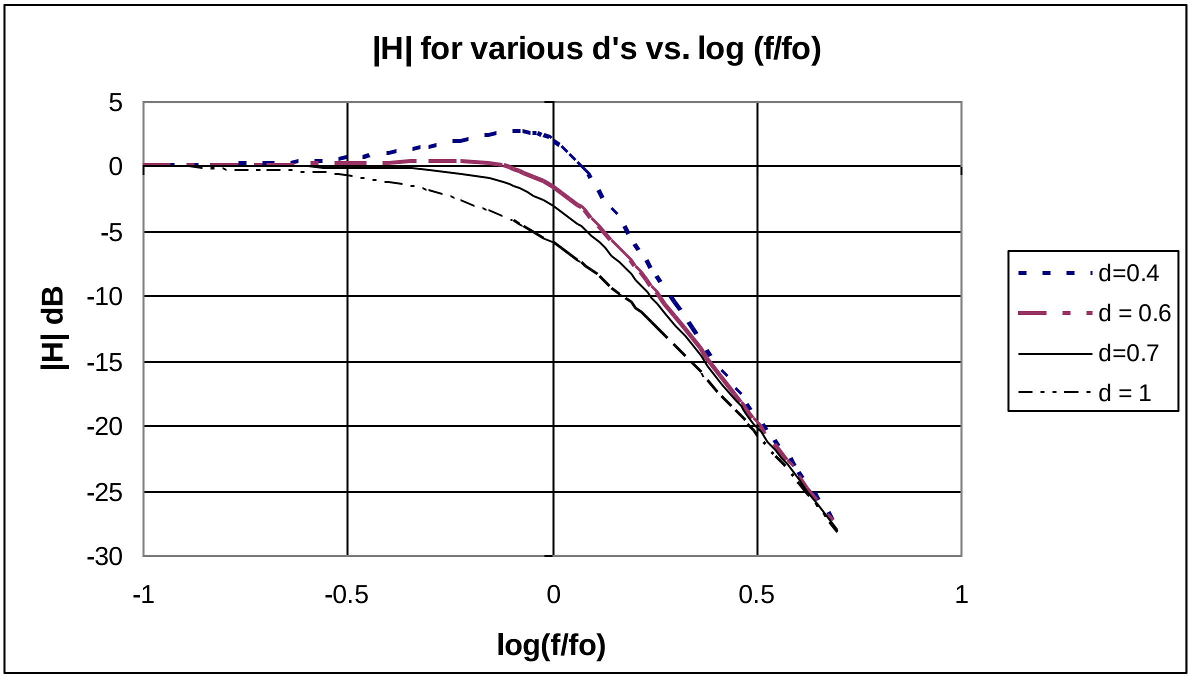



Fig. 3.11

The d's range from 0.4 to 1. Once you get away from the x = 1 region, or $\log(f/f_o) = 0$, they are all similar. For d less than 0.7, the fall off is sharper but you get some overshoot, i.e. $|H| > 1$ near x = 1. I have not shown the phase shift, or phase of H, but you can get a rough idea from eqn. 3.16.

The **high pass** version looks at the voltage across the inductor. I recommend you try to write the magnitude of the transfer function as a function of $x = \omega/\omega_o = f/f_o$ and d. You simply replace $Z_C$ in the **numerator** of first expression in eqn. 3.9 by $Z_L$ and do the algebra. The magnitude of that transfer function should look like the mirror image of the one above, i.e. reflected left to right through $\log(f/f_o) = 0$. For this case $|H_{HP}|$ and $\phi_{HP}$ are

$$|H_{HP}(j\omega)| = \frac{x^2}{\sqrt{(1-x^2)^2 + 4x^2d^2}} \quad \text{and} \quad \varphi_{HP} = \pi - \tan^{-1}\left(\frac{2xd}{1-x^2}\right) \qquad 3.20$$

The **band pass** output would be the voltage drop across the resistor. Here you replace $Z_C$ in the **numerator** of first expression in eqn. 3.9 by $Z_R$ and do the algebra. You should get $|H_{BP}|$ is

$$|H_{BP}(j\omega)| = \frac{2xd}{\sqrt{(1-x^2)^2 + 4x^2d^2}} \quad \text{and} \quad \varphi_{BP} = \frac{\pi}{2} - \tan^{-1}\left(\frac{2xd}{1-x^2}\right) \qquad 3.21$$

The maximum of $|H_{BP}|$ occurs at x =1, or $f = f_o$ and it doesn't depend on d. $|H_{BP}|_{max} = 1$. At that frequency, the phase shift is zero, and the output is the same as the input. However, the sharpness or narrowness of the band pass does depend on the damping factor d. The width of the band pass is defined in terms of the two frequencies where $|H_{BP}| = 1/\sqrt{2}$. If the lower frequency is $f_1$ and the upper is $f_2$, the width is $f_2 - f_1 = \Delta f$. The narrowness of the band pass filter is given by $\Delta f/f_o$ and the Q of the circuit is related to the "narrowness by

$$\frac{1}{Q} = \frac{\Delta f}{f_o} \qquad 3.22$$

I've show a plot of $|H_{BP}|$ for a couple of different d's. Note that as d gets smaller, the range of frequencies that pass with little attenuation becomes smaller and smaller. We say the filter is sharper or narrower. All of the ones shown are narrower than the two-stage RC band pass filter, whose d = 1.

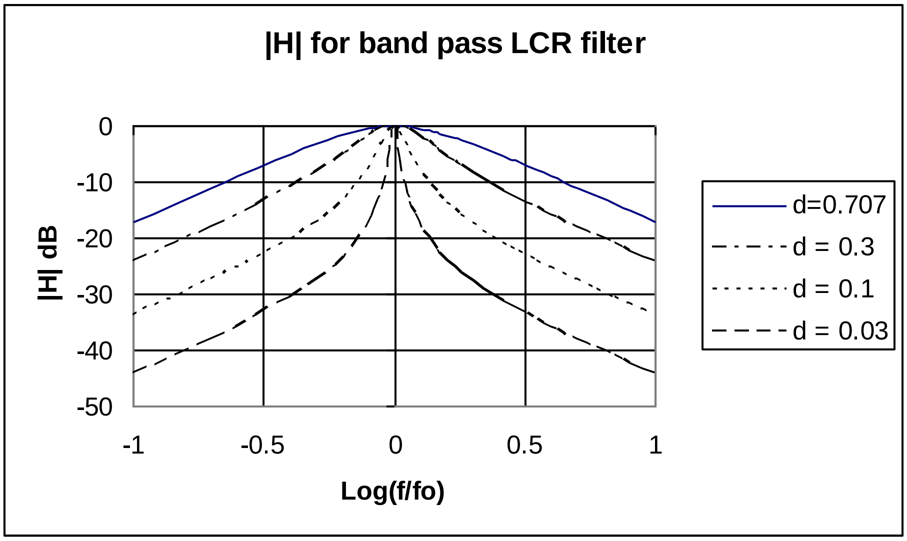



Fig. 3.12

On old radios, the way you selected a station was by turning a dial that changed the capacitance of a variable capacitor that was part of a band pass circuit. This would change $f_o$ to match the frequency of the radio station you wanted to listen to. Then the signal of that station was passed and the signals from other stations were attenuated. In that case you needed a very narrow band pass filter, i.e. a large Q or a small d.

One last note about LCR filters. The voltage across the capacitor and inductor are $180^o$ out of phase, i.e. they have the opposite phase. By arranging the circuit to have the inductor and capacitor together and measure the voltage across both for the output, one gets a **notch** filter. This attenuates the input close to $f_o$, but does not attenuate it far away from $f_o$, just the opposite of a band pass filter. You use notch filters to get rid of interference or noise that occurs at a specific frequency. A common example is a 60Hz notch filter to remove noise picked up from our 60Hz power mains. {Note that the series impedance of an ideal inductor and capacitor, $Z_L + Z_C$ , goes to zero at $\omega_o = 1/(LC)^{1/2}$.}

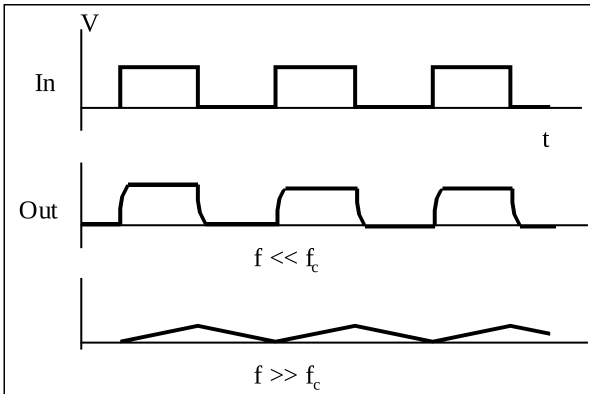
I should warn you that these analyses assume ideal behavior for the resistor, the capacitor and the inductor. Larger inductors, L > 100μH are seldom ideal. They often have 5-20Ω of resistance and some capacitance between the windings. If you take a 10mH inductor and look at the magnitude of its impedance, you will probably find that at low frequencies the ≈ 10-20 Ω of resistance it is likely to have will mean that Z does not →0. Similarly at high frequencies the interwinding capacitance means that its impedance will not continue to increase as the frequency increases, instead at some point it will decrease. Capacitors also exhibit some inductance, but you can usually minimize its effect at frequencies below 1MHz. (This is done by choosing the right type of capacitor. Electrolytic capacitors often have more inductance.) Because it is harder to get good inductors for filters below 1kHz, people often use active filters in the lower frequency ranges. Active filters use operational amplifiers, or op amps for short, resistors and capacitors to "simulate" LCR circuits. We will look at them when we discuss op amps.

I have not discussed parallel LCR circuits; there just isn't time to discuss everything.  You might want to try them on your own.

## E: FREQUENCY DOMAIN AND TIME DOMAIN

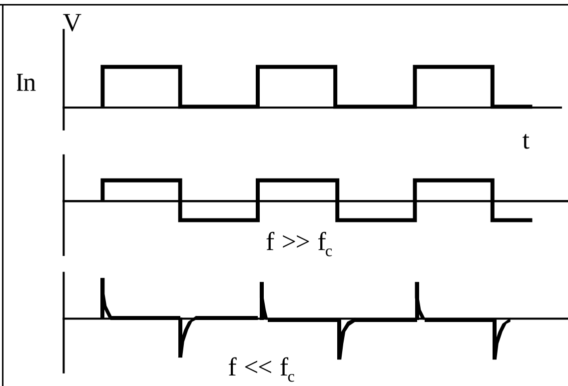
When we describe the response of a circuit to a sinusoidal input, we refer to the description as the **frequency domain** description.  The response is the transfer function and is a function of the frequency of the input signal.  The alternative is a time domain description.  For instance, how would the system respond to a step function input?  Here the response function is a function of time, in particular the time after the application of the step change in the input.  They are two different ways of looking at the same system, and knowledge of one response function will allow us to calculate the other, at least in principle.

It is useful to have an idea of how low and high pass filters respond to a step function input or to a square wave input.  For a low pass filter, if the frequency of the square wave is much less than $f_c$, then it will be passed with little change.  You will merely see a 'rounding of the corners'.  If the frequency is much greater than $f_c$, it will be integrated to look like a triangle wave and it amplitude will be attenuated.  See fig. 3.13.




Low pass filter

Fig. 3.13




High pass filter

Fig. 3.14

If you send a square wave into a high pass filter and its frequency is much greater than the cutoff frequency, it will be passed with little change.  However, the average voltage will be zero, so the swings of the output voltage will be symmetric about 0 volts or ground.  If the input signal varied from 0V to +5V at the input, it will vary from –2.5V to +2.5V at the output.  If the frequency is much less than $f_c$, the output will look like little spikes that occur when the input changes and the spikes quickly decay back to 0V.  The peaks of the spikes would be +5V for the positive one and –5V for the negative one.  See fig. 3.14.