Chapter 2: Time Dependent Signals and AC Circuits

A: Basic RC and RL Circuits

Now we consider circuits where the voltages and currents are not constant, but vary in time. The simplest case is an RC circuit like the one at the right connected to a voltage source where the voltage varies with time. (Note that I'm using E(t) to denote the sources EMF, e.g. a function generator or some such arrangement.) Assume that the voltage has been 0V until t=0 when it suddenly changes to V_o. We still use Kirchhoff's rules to analyze this circuit. At any instant, the sum of the potential changes around the circuit must be zero. (This does not hold for the average value of the voltages, only for the instantaneous voltages!

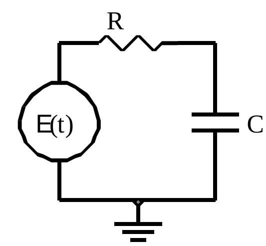


fig. 2.1

This can produce an error if you forget and assume it holds for the average values.) Remember that the voltage drop across a capacitor is $V_C = Q/C$. If we take the current as positive if it flows in a clockwise direction, we have

$$E(t) - IR - Q/C = 0. \qquad 2.1$$

Here Q is the charge on the top plate. We can relate Q to I so that there is only one unknown, Q(t) or just Q. This relation is that I = dQ/dt. Then one has the differential equation

$$E(t) - R(dQ/dt) - Q/C = 0. \qquad 2.2$$

For t<0, E(t) = 0 and Q(t) = 0, and for t>0, E(t) = V_o so for t>0

$$R\left(\frac{dQ}{dt}\right) + \frac{Q}{C} = V_0 \qquad 2.3$$

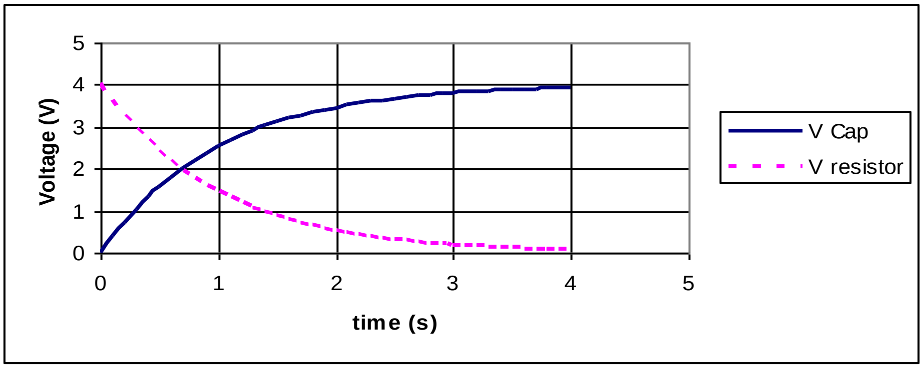
You should verify that the solution to this equation that has Q(0) = 0 is

$$Q(t) = CV_o\,(1 - e^{-t/\tau}) \qquad 2.4$$

where τ = RC has units of seconds if R is in Ohms and C is in Farads. This is the RC charging behavior covered in your basic Physics course. It should be familiar to you and if it isn't, go back and review it. The voltage across the capacitor V_C is just Q/C and the current I = dQ/dt. These are given by

$$V_C = V_o\left(1 - e^{-t/\tau}\right) \qquad \text{and} \qquad I = \frac{V_o}{R} e^{-t/\tau} \qquad 2.5$$

Note that $V_R = IR = V_o\exp(-t/\tau)$. Both of these hold true independent of the order of the two components, R and C, in the circuit. A graph of V_R and V_C is shown below for RC = 1s.



The quantity $\tau = RC$ is called the RC time constant for the circuit. It will show up again when we talk about the behavior of this circuit when $\mathsf{E}(t)$ is a sinusoidal function of time.

A similar result holds for an LR circuit. Here, fig 2.2, an inductor replaces the capacitor in fig. 2.1. The voltage <u>drop</u> across an inductor is $V_L = L(dI/dt)$. Applying Kirchhoff's rules yields the equation

$$\mathsf{E}(t) - IR - L\left(\frac{dI}{dt}\right) = 0 \qquad 2.6$$

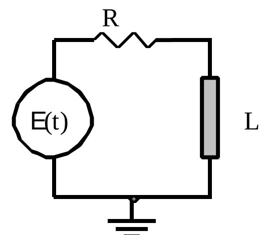


fig. 2.2

If $\mathsf{E}(t)$ has been zero for a long time, I will be zero and dI/dt = 0. Then, if the voltage suddenly changes from 0 to V_0, the equation becomes

$$IR + L\left(\frac{dI}{dt}\right) = V_0 \qquad 2.7$$

for t>0, and I(0) = 0. This is the "same" as eqn. 2.3 and the solution is

$$I(t) = \frac{V_0}{R}\left(1 - \exp\left[-\,{}^{t}\!/_{\tau_L}\right]\right) \qquad 2.8$$

where $\tau_L = L/R$, and is called the LR time constant. In this circuit the I(t) has the same form as Q(t) does in the previous one. The voltages across the resistor, V_R, and the inductor, V_L, are

$$V_R = V_0\left(1 - \exp\left\{-\,{}^{t}\!/_{\tau}\right\}\right) \quad \text{and} \quad V_L = V_0 \exp\left\{-\,{}^{t}\!/_{\tau}\right\} \qquad 2.9$$

All of these are treated in detail in your lower division Physics course. You should be able to figure out what happens if I reverse the situation, i.e. $\mathsf{E}(t) = V_0$ for t<0 and $\mathsf{E}(t)$ changes to 0 at t=0.

<u>B: Input and Output Signals</u>

I can think of the circuit in fig. 2.1 in the following way. I think of applying a voltage or a signal to the two input terminals, the small open circles on the left, and measuring an output voltage or signal across the two output terminals, the two small open circles on the right. In this case the $\mathsf{E}(t)$ represents the input signal and the voltage across the capacitor is the output signal. The resistor and capacitor change the input to produce the

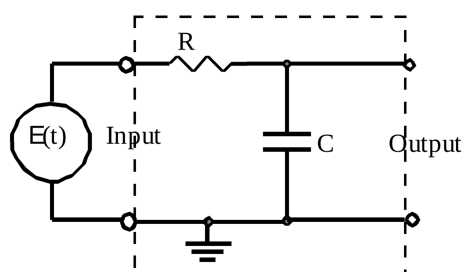


fig 2.3

output signal. That is, they perform a mathematical operation on the

input signal to transform it to the output signal. The objects in the dashed box are the elements that perform the operation. This is a useful way of thinking of the behavior or the function of electric circuits, and much of the time spent in developing electric circuits is spent trying to design ones that perform the particular mathematical operation you want. Note that the input and output signals usually share the same common, so the input signal actually has two connections; one a common or ground and the other a "hot" or signal lead. The output typically has two connections also, the output signal lead and the common lead.

Many complicated E(t)'s can be applied to the input of a circuit. If E(t) has a particular form or shape in a voltage vs. time plot, the output signal may or may not have the same shape. If E(t) has the shape shown at the right, often called a square wave, and it is applied to either the RC circuit in 2.1 or the RL circuit

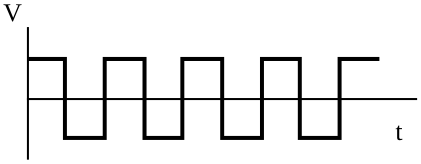


fig. 2.4

in 2.2, the output shape will typically appear to be different than the shape of the input. However, for one particular type of E(t) the shape of the input and the output will be the same. If E(t) = Acos(ωt) where A is independent of time, then the output will be of the form $V_o(t) = B\cos(\omega t + \theta)$. Again B is not a function of time. **The amplitude may change and the phase may be shifted, but the signal is still sinusoidal with the same frequency**. (For sinusoidal voltages like E(t) =Acos(ωt) above, A is the amplitude and is positive while ω is the angular frequency in radians/s. If f or ν is the frequency in Hz, ω = 2πf or 2π ν. If you want the voltage to be equal to – A at t=0, you would write E(t) = Acos(ωt + π), that is, the phase shift of +π would make E(0) = – A.)

C: Sinusoidal Signals

The remarkable result mentioned above occurs because the elements in the circuit, the resistors, capacitors and inductors are all linear elements. This means that the voltage drop across any of these is "linear" in the charge or in the current. By linear, I mean that the voltage drop is determined from the charge or the current by a linear operation, and differentiation and integration are linear operations. These relationships are shown below.

$$V_R = R\frac{dQ}{dt}, \quad V_C = \frac{Q}{C}, \quad \text{and} \quad V_L = L\frac{d^2Q}{dt^2} \qquad 2.10$$

$$V_R = IR, \quad V_C = \frac{1}{C}\int I dt \quad \text{and} \quad V_L = L\frac{dI}{dt} \qquad 2.11$$

If the input in fig. 2.3 above is $\mathcal{E}(t) = A\cos(\omega t)$, then from eqn. 2.1, Kirchoff's rules result in

$$IR + \frac{Q}{C} = A\cos(\omega t) \quad \text{or} \quad IR + \frac{1}{C}\int I dt = A\cos(\omega t) \qquad 2.12$$

where I've used the relations for the current instead of the charge. Because of the integral, the current, I, may not be in phase with the voltage, but it will be a linear combination of cos(ωt) and sin(ωt). In general, Kirchhoff's rules lead to differential equations relating the current or charge to the input voltage. If you are only interested in the **steady state solution**, i.e. the one where E(t) has been equal to A cos(ωt) for a long time, you can convert the differential equations into algebraic ones at the expense of introducing complex numbers. Often we are interested in the steady state solutions so this is a reasonable way to proceed. A good Precalculus course should cover some of the material we will need on complex numbers. Complex algebra is essential to Quantum Mechanics, is used in solving differential equations, and is used to describe wave motion.

D: Complex Numbers

This type of analysis is based on the nature of complex numbers. A complex number z has two parts, a real part and an imaginary part and can be written as

$$z = x + jy \qquad 2.13$$

where x and y are real numbers and j is the square root of – 1. (Since i is often a current, I do not use it for the square root of – 1 in this course.) The real part of z, written Re(z), is x and the imaginary part, Im(z), is y. (Note that **the imaginary part of z is a real number!**)

One often plots z on a graph whose axes are the real axis, or x axis, and the imaginary axis, or the y axis. For instance z = 3 + 4j would be as shown at the right. (Note that 4j = j4; the order does not mater.) The distance of the point from the origin is r and $r^2 = x^2 + y^2$. r is called the magnitude or amplitude of z and written |z|. The angle θ is the phase of z and tanθ = y/x. (Be careful about going from x and y to θ since two different angles between 0 and 360° will have the same tangent.) Also note that

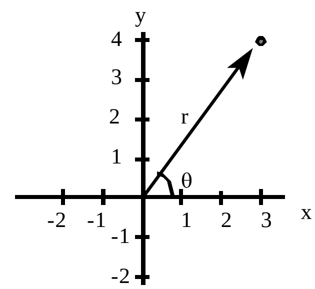


fig. 2.5

$$x = r\cos(\theta) \quad \text{and} \quad y = r\sin(\theta). \qquad 2.14$$

Since the complex exponential, $e^{j\theta}$, is given by

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \qquad 2.15$$

This means that one can write z = x + jy as

$$z = r\,e^{j\theta} = r\cos(\theta) + j\{r\sin(\theta)\} \qquad 2.16$$

and the real part of z is just x = r cos(θ) while the imaginary part is y = r sin(θ). $z = re^{j\theta}$ is called the polar form for z.

One can add, subtract, multiply and divide complex numbers. The rules are as follows. If $z_1 = x_1 + jy_1$ and $z_2 = x_2 + jy_2$, then

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \qquad 2.17$$

If we use the result that $j^2 = -1$, then the product of the two complex numbers is

$$z_1 \times z_2 = (x_1 + jy_1)(x_2 + jy_2) = x_1x_2 - y_1y_2 + j(x_1y_2 + x_2y_1) \qquad 2.18$$

Multiplication and division are easier if you use the polar form, just as addition and subtraction are easier in the standard rectangular form. If $z_1 = r_1\exp(j\theta_1)$ and $z_2 = r_2\exp(j\theta_2)$ then

$$z_1 \times z_2 = r_1r_2\exp(j[\theta_1 + \theta_2]) \qquad 2.19$$

The magnitude of the product is just the product of the two individual magnitudes and the phase of the product is the sum of the two phases. Division is easiest in polar form and

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\exp(j[\theta_1 - \theta_2]) \qquad 2.20$$

The magnitude of (z_1/z_2) is just the ratio of the individual magnitudes and the phase is the difference of the two phases. This also means that the magnitude of the ratio of two complex numbers is just the ratio of the two individual magnitudes. The line below contains several useful expressions.

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \qquad \frac{1}{j} = -j, \qquad j = e^{j\pi/2}, \qquad -j = e^{-j\pi/2} \qquad 2.21$$

Functions of time can also be complex, e.g. $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$. The derivatives of complex functions are taken just like they are for real functions with j acting as a constant; so

$$\frac{d}{dt}\exp(j\omega t) = j\omega\exp(j\omega t), \quad \text{and} \quad \int \exp(j\omega t)dt = \frac{\exp(j\omega t)}{j\omega} \qquad 2.22$$

The nice thing is that the integral or derivative of exp(jωt) is just a constant times the original function.

E: IMPEDANCE

If you are only interested in the steady state solution of circuits containing resistors, capacitors and inductors subject to sinusoidal voltages and currents, then you usually choose to represent the sinusoidal voltages or currents as the real part of a complex exponential, or

$$V_o\cos(\omega t) = \mathrm{Re}\,(V_o\exp\{j\omega t\}) \qquad \text{or } I_o\cos(\omega t) = \mathrm{Re}\,(I_o\exp\{j\omega t\}) \qquad 2.23$$

where V_o and I_o are real constants. If you look at eqn. 2.11 and use the current in 2.23 you will find that you get the same result if you use $I(t) = I_o\cos(\omega t)$ or use $I(t) = I_o\exp(j\omega t)$ and eventually take the real part of the expression after taking any derivatives or integrals. Taking the real part of a complex function is interchangeable with taking the derivative or integral of the function, i.e.

$$\int \mathrm{Re}\{\exp(j\omega t)\}dt = \mathrm{Re}\left\{\int \exp(j\omega t)dt\right\} \qquad 2.24$$

As an aside, this can help you do some integrals quickly. Consider the integrals below. They all give the same result.

$$\int \cos(bx)\exp(-ax)dx = \mathrm{Re}\left\{\int \exp(jbx)\exp(-ax)dx\right\} = \mathrm{Re}\left\{\int \exp([jb-a]x)dx\right\} \qquad 2.25$$

The last integral is much easier to integrate since is it just an exponential where [jb – a] is just treated as any other constant. All you have to do is take the real part of the resulting expression.

This leads directly to the idea of impedance. If the current through an inductor, capacitor or resistor is of the form $I(t) = I_o\cos(\omega t)$, you automatically use $I = I_o\exp(j\omega t)$ with the understanding that you will <u>eventually</u> take the real part of the resulting expression. We often delay taking the real part until the very end. If we use this in eqn. 2.11 we get the results below. **{Note that we have not taken the real part yet**, and $I = I_o\exp(j\omega t)$}

$$V_R = IR, \qquad V_C = \frac{I}{j\omega C}, \quad \text{and} \quad V_L = j\omega LI \qquad 2.26$$

This means that in complex form, the voltage drops are proportional to the complex current. All these expressions have the form V = IZ, where Z is called the impedance and

$$Z_R = R \qquad Z_C = \frac{1}{j\omega C} \quad \text{and} \quad Z_L = j\omega L \qquad 2.27$$

These expressions give the complex impedance for a resistor, a capacitor and an inductor. If we have an inductor where the current is $I(t) = I_o\cos(\omega t) \rightarrow I_o\exp(j\omega t)$, then the voltage across the inductor is given by $V_L = Z_L I = j\omega LI$. But the actual voltage is the real part of this expression. If we put $I = I_o\exp(j\omega t)$, then

$V_L = I_o j\omega L\exp(j\omega t)$ 2.28

but $j = \exp(j\pi/2)$ so

$V_L = I_o\omega L\exp(j\{\omega t + \pi/2\}) = I_o\omega L[\cos(\omega t + \pi/2) + j\sin(\omega t + \pi/2)]$. 2.29

The real part is just the first term, so the actual or real V_L is

$V_L = I_o\omega L\cos(\omega t + \pi/2)$. 2.30

This may seem like a lot of work, but most of the steps will eventually become natural, and this method is especially useful for more complicated circuits. Similarly if the current through a capacitor is $I_o\cos(\omega t) \rightarrow I_o\exp(j\omega t)$, then the complex voltage across the capacitor is just

$$V_C = \frac{I_o}{j\omega C}\exp(j\omega t) = \frac{I_o}{\omega C}\exp\left(j\left\{\omega t - \frac{\pi}{2}\right\}\right) \qquad 2.31$$

and the real part of the voltage is

$$V_C = \frac{I_o}{\omega C}\cos\left(\omega t - \frac{\pi}{2}\right) \qquad 2.32$$

Note that for both the inductor and capacitor the voltage is out of phase with the current. For an inductor, the voltage leads the current by $\pi/2$ and for the capacitor the voltage lags the current by $\pi/2$. (Lead and lag are just expressions and do not refer to any causal relationship.) Similarly for a resistor,

$V_R = I_o R\cos(\omega t)$ 2.33

since the impedance of a resistor is just R, which is real. **When solving circuits subject to sinusoidal currents or voltages, all you have to do is use the impedances to calculate the complex voltages or currents and then take the real part to get the actual ones. (That last step is not usually required for reasons I'll explain later.)** The impedances act just like complex resistances when the applied voltages are sinusoidal, and can be used like "resistances". **That is, the impedances add in series and parallel just like resistors do.** If you have a resistor and a capacitor in series, the equivalent impedance is just $Z_{eq} = Z_R + Z_C$ or

$$Z_{eq} = R + \frac{1}{j\omega C} \qquad 2.34$$

As $\omega \rightarrow \infty$, $Z_{eq} \rightarrow R$ and as $\omega \rightarrow 0$, $Z_{eq} \rightarrow \infty$. At low frequencies the capacitor blocks the current and at high frequencies the impedance of the capacitor is low so the resistance dominates. A capacitor and a resistor in parallel have an equivalent impedance given by

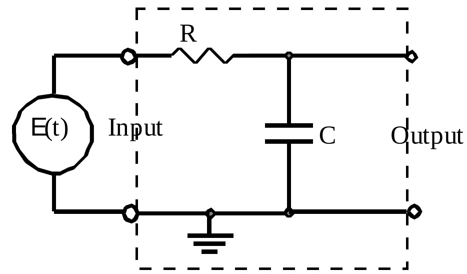
$$Z_{eq} = \frac{Z_R Z_C}{Z_R + Z_C} = \frac{R\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} \qquad 2.35$$

At low frequencies the impedance is approximately R and at high frequencies it goes to 0, because the capacitor lets the current through easily at high frequencies.

You can also use Thevenin's theorem with sinusoidal voltage sources and impedances. Now instead of a Thevenin resistance you have a Thevenin impedance. As a result you think of all voltages sources as having an output impedance, the Thevenin impedance, in series with an ideal voltage source, the Thevenin voltage. For instance, the Thevenin impedance of the circuit in fig. 2.3

is simply the parallel combination of the resistor and capacitor. Therefore it is the Z_{eq} in eqn. 2.35. The Thevenin voltage is just the voltage across the capacitor derived in eqn. 2.37 below.

Consider the circuit in figure 2.3 which I've shown at the right. If $\mathsf{E}(t)$ is $V_o\cos(\omega t)$ you can think of the capacitor as a complex resister. If I want the voltage across the capacitor, i.e. the output voltage, then I just treat the capacitor as a "resistance" = $Z_C = 1/(j\omega C)$. This circuit then looks just like a voltage divider and the output voltage is, using the notation $\mathsf{E}(t) = \mathsf{E}$,



$$V_C = \mathsf{E}\frac{Z_C}{R+Z_C} = \mathsf{E}\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \mathsf{E}\frac{1}{1+j\omega RC} = \frac{V_o \exp(j\omega t)}{1+j\omega RC} \qquad 2.36$$

To get the real or actual voltage as a function of time, one has to take the real part of the expression. It is easiest to do this by writing the denominator in polar form. The magnitude of the denominator is just $\sqrt{1+(\omega RC)^2}$ and the phase is just $\tan^{-1}(\omega RC)$. Therefore

$$V_C = \frac{V_o \exp(j\omega t)}{\sqrt{1+(\omega RC)^2}\ \exp\left(j\tan^{-1}\{\omega RC\}\right)} = \frac{V_o \exp\left(j\{\omega t - \tan^{-1}(\omega RC)\}\right)}{\sqrt{1+(\omega RC)^2}} \qquad 2.37$$

The magnitude of the output is proportional to the magnitude of the input voltage but multiplied by

$$\frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(\omega\tau)^2}} \qquad 2.38$$

where $\tau = RC$, the familiar RC time constant. The frequency of the output voltage is the same as the input but the phase is shifted by $-\tan^{-1}(\omega RC)$.

In general the steady state output voltage in circuits like this will have the <u>same frequency</u> as the input voltage but the phase may be shifted. Also the output amplitude will be proportional to the input amplitude, but multiplied by a constant (i.e. <u>no time dependence</u>) that depends on the R's, C's and L's in the circuit and on the frequency ω.

When analyzing circuits we're usually interested in these two constants, the phase shift and the constant that scales the output amplitude relative to the input amplitude. This leads us to the concept of a transfer function.

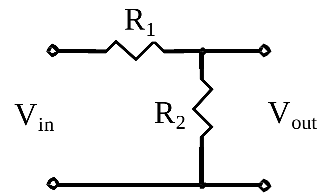
F: TRANSFER FUNCTIONS

In circuits where all the elements in the circuit are linear, i.e. the voltage drop across any element is a linear function of the current, and where the voltage sources are sinusoidal, the complex voltage across any element in the circuit is a linear function of those voltage sources. If we have one source, the input voltage, and are interested in the voltage across a particular element, which we'll call the output voltage, the output voltage with be equal to this input voltage times a complex constant, H, that depends on the resistances, capacitances and inductances in the circuit and on the frequency. Therefore the complex output voltage (before we've taken the real part) is of the form

$$V_{out} = V_{in}H(j\omega) \qquad 2.39$$

where we usually write H(jω) to make sure you realize it depends on the frequency and may be complex. H(jω) is just V_{out}/V_{in}. Since V_{out} is proportional to V_{in}, they have the same time dependence, $e^{j\omega t}$, and therefore the same frequency. (Some people use the notation H(ω).)

Consider a simple voltage divider circuit like the one at the right. In this circuit the output voltage is given by



$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

so the transfer function is just the divider ratio of $R_2/(R_1 + R_2)$.

In this case, just resistors, H is real and not complex and not a function of frequency. However, if you have capacitors and/or inductors in the circuit, H may be complex and a function of the frequency.

If you look at the example above (from fig. 2.3), the output voltage is the voltage across the capacitor, V_C, and the input voltage is V_{in} = E = $V_o\exp(j\omega t)$. This is just a voltage divider with impedances instead of resistances. From eqn. 2.36 one can see that the transfer function is just

$$H(j\omega) = \frac{Z_C}{Z_R + Z_C} = \frac{1}{1 + j\omega RC} = \frac{\exp\left(-j\tan^{-1}[\omega RC]\right)}{\sqrt{1+(\omega RC)^2}} \qquad 2.40$$

If I choose to write H(jω) in polar form, it will look like

$$H(j\omega) = |H(j\omega)| \exp(j\phi) \qquad 2.41$$

Where ϕ is just some real constant called the **phase shift**. Thus when I multiply V_{in} by H(jω) in polar form I have a particularly simple looking result,

$$V_{out} = V_{in} H(j\omega) = V_o\exp(j\omega t)\bullet|H(j\omega)|\exp(j\phi) = V_o| H(j\omega)| \exp(j\{\omega t + \phi\}) \qquad 2.42$$

Thus if V_o is real, as we usually take it to be, then V_{out} is already in polar form with an amplitude or magnitude

$$|V_{out}| = V_o| H(j\omega)| \qquad 2.43$$

and a phase given by

$$\text{Phase of } V_{out} = \omega t + \phi \qquad 2.44$$

Therefore when I want the actual output voltage, or the real part of V_{out}, it will just be the amplitude times the cosine of the phase of V_{out}.

$$\mathrm{Re}(V_{out}) = V_o| H(j\omega)| \cos(\omega t + \phi) \qquad 2.45$$

Thus if we can determine H(jω) in polar form, we can simply write down the output voltage. Therefore we usually try to figure out the transfer function for the circuit, H(jω), since it will quickly give us the output voltage for ANY sinusoidal V_{in}. In fact **we usually characterize a circuit by its transfer function**. The next chapter will be devoted to finding the transfer functions for different circuits.

One final note is that these circuits are linear. If my input consists of two (or more) sinusoidal inputs, say the complex $E_{in}(t) = V_{o1}\exp(j\omega_1 t) + V_{o2}\exp(j\omega_2 t)$, and we know the transfer function H(jω), then the complex output voltage is just

$$V_{out} = H(j\omega_1)\bullet V_{o1}\exp(j\omega_1 t) + H(j\omega_2)\bullet V_{o2}\exp(j\omega_2 t) \qquad 2.46$$

The output voltage for a sum of inputs is the sum of the individual outputs. Therefore if an input voltage can be represented as a sum of sinusoidal inputs, we can find the output if we know the transfer function for an arbitrary sinusoidal input. Knowing H(jω) allows us to find the steady state output for any input that can be represented by a Fourier series or a Fourier transform.

G: Energy and Power

The instantaneous power, or rate of work being done on a device is equal to the current flowing through the device times the voltage drop across the device,

$$P(t) = \frac{dW}{dt} = I(t)V(t) \qquad 2.47$$

This expression is valid for linear devices such as ideal resistors, capacitors and inductors; and for non-linear devices like diodes and transistors. If V and I do not change, P is constant. However, if they depend on time the power is not constant. We often calculate the average value of the power over a long time, or

$$P_{ave} = \lim_{T\to\infty} \left\{ \frac{1}{T} \int_T I(t)V(t)dt \right\} \qquad 2.48$$

For periodic signals this amounts to averaging the instantaneous power over one cycle. For sinusoidal signals, $P_{ave} = 0$ for an inductor or capacitor because the current and voltage are "out of phase by 90^o", e.g. the average of sin(ωt) cos(ωt) over one cycle = 0. For a resistor, I = V/R, so

$$P_{ave} = \frac{1}{\text{period}} \int_{\text{oneperiod}} \frac{V^2}{R} dt \equiv \frac{V_{rms}^2}{R} \qquad 2.49$$

where the identity comes from defining $(V_{rms})^2$ as the average of V^2 over one cycle. For a sinusoidal voltage, V = Acos(ωt), $V_{rms} = A/\sqrt{2} = 0.707A$. For a square wave that oscillates from +A to –A, V_{rms} = A. You might try to calculate V_{rms} for a triangle wave that goes from +A to –A. (I've been brief here because you should know most of this from your lower division Physics class.)

Warning

Calculating the energy or power in an electric circuit is a nonlinear operation involving the current and/or the voltage, e.g. for a resistor the instantaneous power is V^2/R or I^2R. Because of this, ***you MUST take the REAL PART of an expression involving complex voltages or currents BEFORE multiplying them together***. The concept of impedance and complex voltages and currents works because, for resistors, capacitors and inductors, the relationships between the voltages and currents are linear. ***In that case the linear operations and the fact that R, C and L are real quantities, means that you can do all the operations and manipulations before taking the real parts. That is, the order of taking the real parts and the other mathematical operations are interchangeable. Energy and power involve nonlinear operations, and the order of taking the real parts is not interchangeable with these mathematical operations.***