ADVANCED PHYSICS LABORATORY

Lab 1: Using a Pendulum to Measure g (Due September 6)

In this laboratory I want you to use a simple pendulum to determine g, the local gravitational field. Try to measure g to an accuracy of at least 0.1%. By this I mean that your calculated or estimated uncertainty is less than 0.1%, not just that your measurement is within 0.1% of the accepted value of g. Make a careful, quantitative estimate of the uncertainty in your measurement.

Your lab report must contain the following sections or discussions and you should follow this pattern for this lab only, i.e. the Pendulum Lab.

1. An introduction that explains what you are measuring and the general theory behind the technique you are using; in this case how g is related to the physical parameters of your pendulum. I want you to explain how the equation of motion for the pendulum simplifies to the result you will use to calculate g from your measurements. You need identify any approximations and **show** they will not affect your results by more than 1 part in a thousand. This should include the following
   a) The small angle approximation.
   b) The point mass approximation.
   c) Neglect of damping.

   The relationship between g, L and T for a simple pendulum is usually given as,

   $$T = 2\pi\sqrt{\frac{L}{g}}\,, \qquad (1.)$$

   in lower division courses. This assumes that the mass is a point mass, that there is no damping, that the mass of the string is negligible, and that the amplitude of the oscillation is small. If you want to use this relation, you must show that these approximations are valid for your measurement parameters.

2. An experimental methods section that explains how you actually made the measurement. This should include enough detail so that someone else could reproduce your measurements from your description. Diagrams are useful here. (You should choose parameters for your measurement, e.g. the length of the pendulum, so that you will minimize the uncertainties and errors due to approximations.) Be sure to identify which quantities you are measuring and how.
3. A results section that presents your data and the calculation of g from your measured quantities. You can present your data as a spreadsheet, but be clear and concise. Label quantities! In the case of Method I, you should present your analysis of the accuracy of your measurements in terms of the propagation of the errors in your measured quantities to your calculated g, and I want to see how you do the calculation. The most obvious uncertainties include the measurement of the length of the pendulum and the period of the pendulum. However, it should also include a quantitative discussion of how any approximations you have made will influence your results. This should consider the effects of the point mass approximation, the small angle approximation, and the neglect of damping.
4. An analysis that discusses the significance of your results. For the Pendulum Lab this section will probably be short.

One purpose of this first lab is to get you to make a careful, laboratory report. You should have to do at least one such report. Another purpose is to make you calculate the uncertainties in your measurements, i.e. make quantitative estimates of your uncertainties, especially in Method I.

The report does not have to be long if you are concise. Just make sure you address the above points.

Experimental Method I

There are two ways to measure g with a pendulum. The most common one is the use equation 1 and measure the period and the length and calculate g from those measurements for at least two different lengths. Here the length is measured from the point of suspension to the center of mass of the ball. **This is the method you will use for this lab!**

~~Experimental Method II~~

~~A second method is to rearrange equation 1 to the form~~

$$T^2 = 4\pi^2 \frac{L}{g} \qquad 2.$$

~~Now you can measure the period for various L's and plot T^2 vs L. Here the slope is $4\pi^2/g$. This can be useful if you aren't sure where the center of mass of the small weight is. Since T^2 is proportional to L, L can be off by a fixed constant every time and still produce the same slope. In this case L could be the distance from a fixed point on the suspension device to a fixed point on the small weight (not necessarily the center of mass) and give the correct slope and hence the correct g. **I want you to try both methods** for measuring g and compare them. (You will need to vary L enough to be able to get an accurate slope.) This method is the best when it is difficult to estimate the position of the center of mass.~~

Before you start Method I want you to make two calculations.

1. Assume that you have a pendulum of length L=1m. How accurately would you have to measure L for your calculated g to be within 0.1% of the actual value? (Assume equation 1 is accurate and all the uncertainty is in the length.)
2. Repeat this assuming all the uncertainty is in the period measurement and find how accurately you need to measure T if all the uncertainty is in the time measurement.

Equipment

You will probably need to use the tape measures to measure the length. Use the ones with steel tapes. You can use the Pasco photogate timers to measure the period of the pendulum. (Usually I would ask you to do one by hand with a stop watch, but you don't have to.) If you use a photogate, make sure the ball goes through the detecting beam; the string is usually too thin to trigger it reliably. If you use the Pasco timers, you only need 15 to 20 periods. (The first period is not always reliable, so I recommend you don't use it. Also you should also time it with a stop watch just to check it.)

Appendix A: Amplitude and Damping Dependence

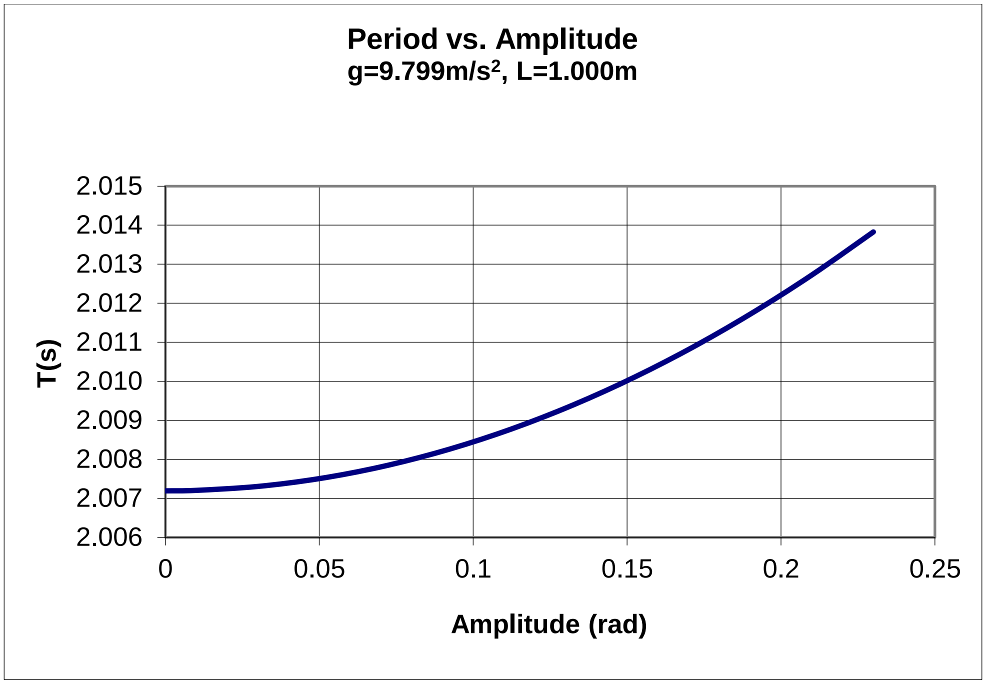
The usual relationship between g, L and T for a simple pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}}\,, \qquad \text{A1.}$$

assumes that the mass is a point mass, that there is no damping, that the mass of the string is negligible, and that the amplitude of the oscillation is small. If the amplitude is not small, you can determine the relationship by doing an integral, an elliptic integral. For small angles of vibration, θ, the integral can be expanded in a power series yielding the relationship for the first two terms in the series,

$$T \approx 2\pi\left[\sqrt{\frac{L}{g}}\right]\left(1+\left(\frac{\theta_{\max}^2}{16}\right)\right). \qquad \text{A2.}$$

where θ_{max} is the maximum value of θ. Note that this reduces to the original equation when θ_{max} goes to 0. The next term involves $(\theta_{max})^4$, and should be negligible for your purposes. (Remember that **θ is measured in radians**.) I have shown below how the period varies with amplitude for a pendulum of length 1.000m and a $g=9.799 m/s^2$.



Damping also shifts the frequency of oscillation for free vibrations for a simple harmonic oscillator. If ω_o is the natural frequency and γ represents the damping, then the frequency of free vibrations is related to ω_o and γ by

$$\omega^2 = \omega_o^2 - \gamma^2. \qquad \text{A3.}$$

It is the undamped natural frequency, ω_o, that is given by $\omega_o = \sqrt{g/L}$. Can you think of a way to measure the damping, or at least put an upper limit on γ, and show that it has a very small effect on ω? Note that the amplitude of vibration for a harmonic oscillator has the form

$$\theta(t) = \theta_o \exp\{-\gamma t\}\cos(\omega t + \phi) \qquad \text{A4.}$$

The amplitude of oscillation will "decay" by a factor of $e^{-\gamma t}$, so you can put an upper limit on γ by watching the decay of the vibrations. (You may want to go back to your Classical Mechanics or General Physics texts and look at damping and simple harmonic motion.)

> For instance if the amplitude of oscillation, $\theta_o\exp(-\gamma t)$, decays to ½ of the original value, i.e. $\theta_o/2$, in 5 cycles, then
>
> $\theta_o\exp(-\gamma t) = \theta_o/2$, or $\exp(-\gamma t) = 0.5$
>
> when $t = 5T = 5(2\pi/\omega)$. This means that, taking natural logs,
>
> $-\gamma(10\pi/\omega) = -0.7$
>
> or
>
> $\gamma \approx \omega/(14\pi)$.
>
> This means that $\gamma^2 \approx \omega^2(5\text{x}10^{-4})$. From this you should be able to figure out how much ω differs from ω_o, i.e. you should be able to find the fractional difference, $(\omega_o - \omega)/\omega_o$ and convert it to a percentage.

Appendix B Propagation of Errors

To remind you about propagation of errors, I will work out an example for you. You will need to apply it to your experiment.

If I want to estimate the value of π by measuring the circumference, C, and diameter, d, of a circle, I will ***calculate*** π by

$$\pi = C/d \qquad \text{B1.}$$

If d and C and have mean values, <d> = 10.01 cm and <C> = 31.15cm, and standard deviations of the **mean**, $\sigma_d = 0.2$ cm and $\sigma_C = 0.3$cm, for both, what is my best estimate of π and the standard deviation of π, σ_π? The best estimate of π is to use <d> and <C> in 1, π = <C>/<d>= 3.112

Estimating σ_π from σ_d and σ_C is a little more complicated; so I'll just give you the rules. If I have a quantity g as a function of x and y, g = f(x,y), then my best estimate of g is $g = f(<x>,<y>)$. That is, I just use the mean values of x and y in f(x,y) to calculate g. The standard deviation of g, σ_g, is given by

$$\sigma_g^2 = \sigma_x^2 \left(\frac{\partial f}{\partial x}\right)^2 + \sigma_y^2 \left(\frac{\partial f}{\partial y}\right)^2 \qquad \text{B2.}$$

where the partial derivatives are to be evaluated at the mean values of x and y. This is an approximation that works well if f(x,y) does not vary too rapidly about the point $(<x>,<y>)$.

If we apply this to the problem above, $\pi = f(d,C) = C/d$ and

$(\partial f/\partial d) = - C/d^2$

$(\partial f/\partial C) = 1/d$

Inserting these into eqn. 2 yields

$(\sigma_\pi)^2 = (\sigma_d)^2(- C/d^2)^2 + (\sigma_C)^2(1/d)^2$

or

$(\sigma_\pi)^2 = (0.2\text{cm})^2(- 0.31/\text{cm})^2 + (0.3\text{cm})^2(0.1/\text{cm})^2$.

Calculating σ_π yields a value of 0.069. As a result our best estimate of pi is $\pi = 3.112 \pm 0.069$. Note that in this case, π and σ_π are dimensionless. In many cases means and standard deviations will have dimensions. **Be sure you indicate the dimensions, if there are any**. (It is customary to round the standard deviation to two significant figures and round the calculated value of π to the same level of "significance". If the standard deviation is rounded to the thousandths also round π to the nearest thousandth.)

A result of this method is that if f is the fractional standard deviation, i.e. $f_g = \sigma_g/<g>$, then for a product of terms where x, y, and z are the variables and a, n, and m are constants and z = z(x,y) such that

$z = ax^n y^m$

then

$f_z^2 = (nf_x)^2 + (mf_y)^2$

This is often the easiest form to use for a product of terms.

You will need to apply these same methods to estimate the uncertainty in g from the uncertainties in your measurements of the period and length of the pendulum.