Advanced Lab

Faraday Rotation and the Verdet Constant

In this lab should make the two measurements below.

1. Verify Malus's Law by measuring the intensity of transmitted light as a function of angle for the polaroid. This is equation 2 below.
2. Measure the Verdet constant of water using light from the red, yellow, green and blue diodes. There are book values for light of various wavelengths. Check the web page for the course. You should compare your measured values to the "book" values.

I. Polarization and Malus' Law

When linearly polarized light is passed through a Polaroid with the preferred direction of the Polaroid making an ϑ with the E of the polarized light, the intensity, I, is reduced by a factor of

$$\cos^2(\vartheta)\ . \qquad 1.$$

If I_o is the intensity for $\vartheta = 0$, then

$$I(\vartheta) = I_o\cos^2(\vartheta)\ . \qquad 2.$$

Note that I_o should be measured with the Polaroid in place, since the Polaroid may reduce the intensity of the beam even if $\vartheta = 0$. (I will use the lower case i for the current!) This is called Malus's Law

If the Polaroid and the polarization of the beam are initially at $\vartheta_o = 45^o$, $\pi/4$, and the angle is changed by a small amount $\Delta\vartheta$, then the intensity will be given by

$$I(\vartheta + \Delta\vartheta) = I_o\cos^2(\vartheta_o + \Delta\vartheta). \qquad 3.$$

Using the appropriate trigonometric identities and a small angle approximation, equation 3 becomes

$$I(\vartheta + \Delta\vartheta) \sim \frac{I_o}{2}\,(1 - 2\Delta\vartheta). \qquad 4.$$

For this special case, $\vartheta_o = \pi/4$, the change in intensity is proportional to the change in angle, for small changes. (Note that ϑ is in radians for this approximation.)

II. Faraday Rotation

It has been found that the application of a magnetic field may cause materials to rotate the plane of polarization of a light beam that passes through the material. This is called Faraday rotation.

If the magnetic field, B, is constant and the light beam traverses the material parallel to B, then the rotation is proportional to B and the thickness of the material. If the Rotation is $\Delta\vartheta$, then this is written as

$$\Delta\vartheta = V_B BL \qquad 5.$$

where V_B is the Verdet constant, B is the magnetic field and L is the distance the light travels through the sample. This assumes the direction of propagation is parallel to B.

Our samples will be inside a solenoid and the B field will be produced by sending a current i through the coils of the solenoid. Then

$$B = \mu n i \qquad 6.$$

where n = turns per meter for the solenoid and μ is the magnetic permeability of the material. (This is only accurate near the center of the solenoid. At the end, B drops to half of this value.) Comparing equations 4, 5 and 6, one can see that the change in intensity due to the rotation produced by the magnetic field is

$$\Delta I = -I_o V_B B L = -I_o V_B L \mu n i. \qquad 7.$$

If we know μ and can measure i, n, L and $\frac{\Delta I}{I_o}$, then we can calculate V_B. The number of turns in the coil, N, is given, so $n = N/L_{coil}$. (Are L and L_{coil} the same for our samples?) The current in the coils can be measured with an ammeter and μ can be "looked up." For most of our materials $\mu \approx \mu_o$. The only difficulty is measuring $\frac{\Delta I}{I_o}$.

We will measure the light intensities with a photodiode. A reversed biased photodiode will allow a current to flow through it that is proportional to the power of the light striking it. The constant of proportionality depends on the wavelength of the light, but not on the intensity, i.e. the relation to the intensity is linear. This current is converted into a voltage by running it though a resistor and the voltage is proportional to the power of the light hitting the diode. If V_o is the voltage produced when $\vartheta = 0$ and ΔV is the change in voltage produced by running a current through the solenoid with $\vartheta = \pi/4$, then

$$\frac{\Delta I}{I_o} = \frac{\Delta V}{V_o} \quad . \qquad 8.$$

V_o can be measured with a voltmeter. Adjust the Polaroid so that the voltage is a maximum, this will occur when $\vartheta = 0$ and gives V_o. Then rotate the Polaroid so that the measured voltage is one half this value. This should occur when you have rotated the Polaroid 45^o from the position it had when the voltage was a maximum. Now apply an oscillating current to the solenoid and measure the amplitude of the current and the amplitude of the oscillating voltage it produces. The amplitude of the current will be the i in equation 7 and the amplitude of the voltage will be the ΔV in equation 8. **One problem is that room lights may contribute to V_o and therefore to $V_o/2$. If it does it, it will make your Verdet constant too small.** Can you account for this?

Example:

If the maximum voltage, V_o = 4V, and for a measured current of i = 10 mA the ΔV = 0.1mV, then the Verdet Constant V_B is given by

$$V_B = \frac{\Delta V}{L \mu n i V_o}. \qquad 9.$$

If nL = 960 turns and $\mu = \mu_o$, then V_B = 2.07 rad/T•m. In older books V_B is given in minutes per Gauss (Orstead) per cm. Then this V_B would be 0.007 min/G•cm. At λ = 596nm and T = 20ºC, water has a Verdet constant of 13.1×10^{-3} min/(G•cm).

III. Lockin Amplification

Voltage changes of < 1mV are often hard to measure reliably, especially when the change is from 1.105V to 1.106V. Here the fractional change is one part in 10^3. Drift and other noise will often obscure such small changes, especially when the signal is superimposed on a large background. One way to overcome this problem is to modulate the signal so that it can be separated from the background. For the Faraday rotation one can make the current i that produces the magnetic field oscillate at some frequency ω by driving the coils of the solenoid with a function generator. Then ΔV will oscillate at this same frequency (if $\vartheta_o = \pi/4 = 45^o$). One can then look for voltages at this frequency and separate them from the drift and the noise at other frequencies.

There are several ways to measure voltages in a particular frequency range. One way is to construct a simple bandpass analog filter (RCL type). A more powerful way is to use a lock-in amplifier. A lock-in amplifier measures the component of a signal at a particular frequency, just like measuring a Fourier component of a signal. If the voltage (signal) is a function of time, V(t), and the lock-in is given a reference signal at a frequency ω_r, the lock-in will measure the component of V(t) at the frequency ω_r of the reference. It does this by multiplying V(t) by $\cos(\omega_r t + \phi)$ and averaging the result. The phase of the multiplying signal ϕ is adjustable. The mathematics of this is fairly straightforward. If $V(t) = A\cos(\omega t)$, the lock-in multiplies it so that the resultant product is

$$V(t)\bullet\cos(\omega_r t+\phi)=A\cos(\omega t)\bullet\cos(\omega_r t+\phi) \qquad 10.$$

which can be written as

$$(\ \ )\bullet\{ \cos([\omega - \omega_r] t - \phi) + \cos([\omega + \omega_r] t + \phi) \} \qquad 11.$$

If this is averaged (integrated) for a time $T >> 1/\omega$ or $1/\omega_r$ (a low pass filter with T = RC) then the second term will be multiplied by a factor of

$$\frac{1}{T(\omega+\omega_r)} << 1. \qquad 12.$$

This will make the second term negligible. If $[\omega - \omega_r]T > 1$, the first term will also be attenuated. However if $\omega = \omega_r$, then the first term is not attenuated and it is given by

$$\left(\frac{A}{2}\right)\cos(\phi) \qquad 13.$$

If one averages for a long time the only term left at the output will be the one in eqn. 13. Thus you will have measured the amplitude of V(t) that oscillates at a frequency $\omega = \omega_r$. If one adjusts the phase of the reference channel so that $\phi = 0$, then the output is A/2. The lockin scales this so that the output will be the RMS value (A/). Some lockin amplifiers will find both the magnitude and the phase so that you do not have to adjust the phase to maximize the output.

Some lockins actually multiply the V(t) by a square wave at the reference frequency instead of a sine wave because it is easier and more accurate. The main difference is that the components of V(t) at the odd harmonics of the reference also will show up at the output. Our lockin does not do this.

IV Calculating BL

The term BL, the magnetic field times the distance the light travels through the field in that medium should really be written as

$$\int B(x)dx \qquad 14.$$

where one integrates from the beginning of the material to the end. If B is constant, then the integral is just BL. However, B is not constant. One can calculate B on the axis of a solenoid as the superposition (sum) of B's due to loops or rings. The B on the axis of a ring of radius R and a distance x from the center of the ring is

$$B(x) = \frac{\mu I d^2}{2\left(x^2 + R^2\right)^{3/2}} \qquad 15.$$

The field from a solenoid of length L with n turns per unit length and a current i_o (per turn) can be approximated by breaking it into rings of length dx and current $ni_o dx$. Then the dB at x_o due to the loop at x is

$$dB = \frac{n a^2 \mu i_o d x}{2\left((x - x_o)^2 + R^2\right)^{3/2}} \qquad 16.$$

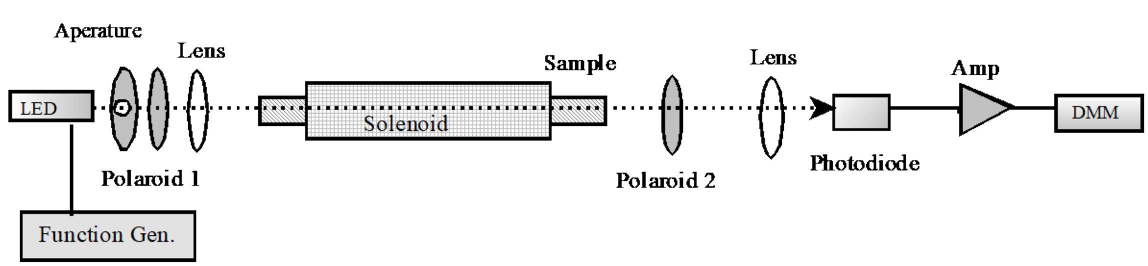
One can integrate this from x_1 to x_2.to get the field at x_o.

For our set-up, $V_B = 410*\Delta V/(i*V_o/2)$, if Δv is in mV, i is in mA and $V_o/2$ is in Volts.

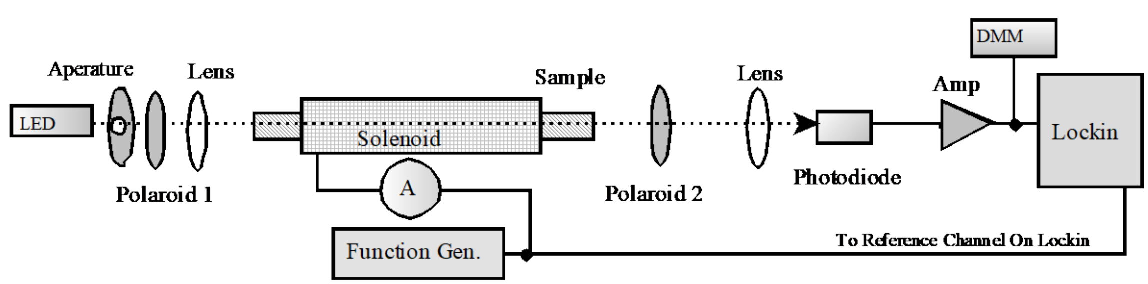
V. Experimental Set Up

The equipment you need is listed below.

1. A laser or LED to produce the light
2. A solenoid and function generator to produce the magnetic field
3. A tube containing the sample (water) that you will put inside the solenoid
4. Two polaroids
5. A photodiode and current to voltage converter to detect the light.
6. A lock-in amplifier to detect the signal
7. Two digital multimeters (DMM), one to measure voltage and one for current
8. Two lenses (if you use the LEDs)



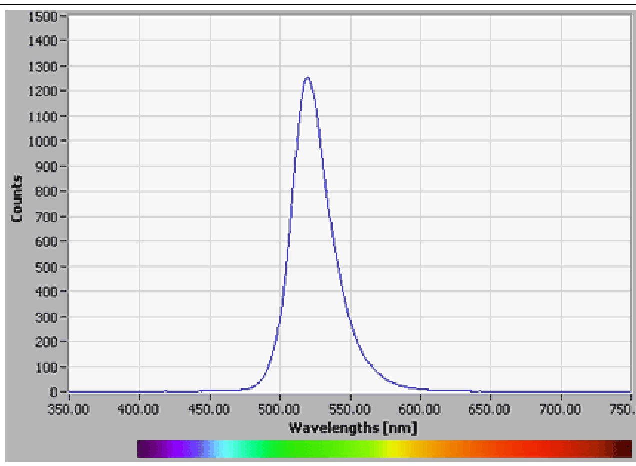
Setup for Measuring Malus' Law



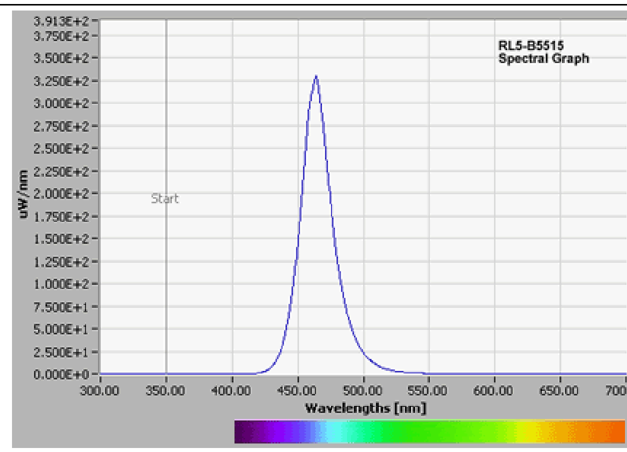
Setup for Measuring the Verdet Constant using an LED

The first diagram shows the setup for measuring Malus' Law. Here you will drive the LED with the function generator with a signal of a few hundred Hz and use the AC Voltage function on the DMM to measure the signal. This will eliminate the effects of the room lights whose intensity does not vary in time. You will probably need the lenses to focus the LED beam on the detector. You could do this without the solenoid and sample, but you might as well just leave it in place; it won't affect the results for Malus' Law. **You will need to align the system so that the light from the LED goes down the center of the solenoid.** You want to avoid having the light scatter off the sides of the tube. (The aperture can help eliminate light that is not going to go down the center of the sample.) Center the sample in the solenoid. Measure the intensity with the Polaroids aligned and then keep changing the angle by 10^{o} and record the intensities. Go through about 180^{o}. Plot intensity vs angle and intensity vs $\cos^2$ of the angle. Does it follow Malus' Law?

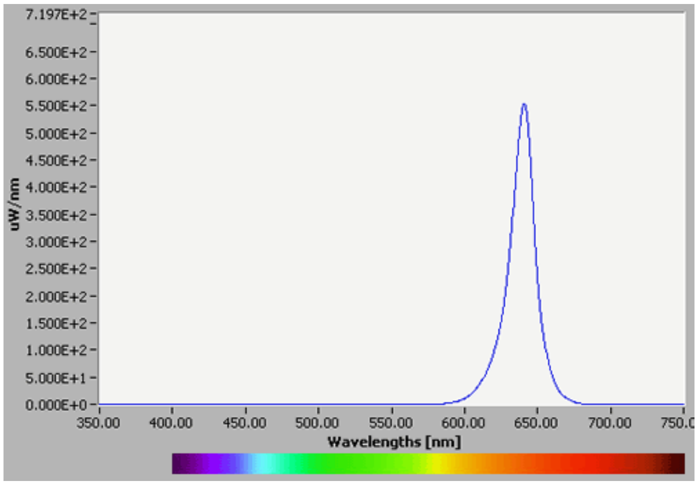
To measure the Verdet constant you will use several different colored LEDs. Here you will drive the LED with a DC power supply and use the function generator to drive the solenoid. You will need to measure the current through the solenoid. Again, use a frequency of about 210Hz, but one that is **NOT** a multiple of 60Hz. The signal from the generator also goes to the reference channel of the lockin. You need the quantities that go into equations 7 and 8. First measure V_o with the polaroids aligned; this will be the maximum voltage on the DMM (DC voltage) connected to the signal from the amplifier. (Room light will interfere with this reading, so you should use the "hood" on the photodiode and keep the room lights low.) Magnetic interference from the solenoid (dB/dt effects) can produce a spurious signal, so the photodiode and amplifier should not be too close to the solenoid. Now adjust the polaroids so that they are at 45^{o} to each other; the DMM should read $V_o/2$ at this point. Here the lock-in will read ΔV in equation 8. Measure this for several currents in the range of 20mA to 80mA. Then switch the LED to do this for a different wavelength. Do this for red, yellow, green and blue LEDs. How accurate are your measurements? How does the Verdet constant vary with wavelength? I have shown the spectra for the different LEDs given by the manufacturer.



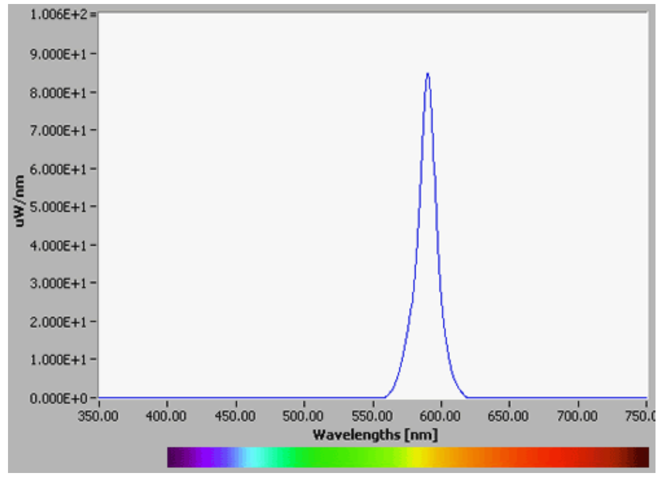
Green LED Spectra



Blue LED Spectra



Red LED Spectrum



Yellow LED Spectrum