Simple Pendulum (Method II)

Introduction

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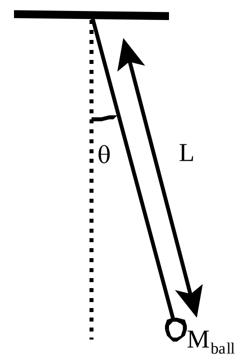
We have measured the local gravitational field, g, using a pendulum shown at the right. The pendulum consists of a lead ball of mass M_b at the end of a string. The string is clamped at the top and the distance from the point where it is clamped to the center of the ball is L. θ is the angle between the string and a vertical line from the clamp. The motion of the pendulum is governed by the relation



$$I\alpha = \tau \qquad 1.$$

where I is the moment of inertia of the system about an axis through the clamping point, α is the angular acceleration and τ is the torque.

If the mass of the ball is much larger than the mass of the string, the angle θ is small, and the radius of the ball is much less than L, then the ball acts like a point mass and the motion is simple harmonic and the period of the motion is given by

$$T = 2\pi\left[\sqrt{\frac{L}{g}}\right] \text{ or } T^2 = 4\pi^2 \frac{L}{g} \qquad 1.$$

In our case radius of the ball was 0.017m and the smallest L is 0.94m so L was greater than 50 times the radius of the ball and the θ's used were < 0.07 radians. The mass of the ball was 0.234kg and the mass of the string (fishing line) was ≤ 0.00015kg. As a result equation one should be good to better than 0.1%.

The major difficulty is the experiment is accurately measuring the distance to the center of mass of the ball, i.e. L. We can get around this if we can measure accurately to a given, fixed point on the ball and varying this distance. Call this distance L*. Then L = L* + δ where d is a fixed distance from the chosen point on the ball to the center of mass. For instance L* could be the a fine line drawn on the ball, as long as we measure to the <u>same</u> fixed point each time. Then equation one becomes

$$T^2 = 4\pi^2 \frac{L* + \delta}{g} \qquad 2.$$

If we measure T for several lengths, L*, and plot T^2 vs L* the slope should be $4\pi^2/g$. This would allow us to calculate g from that slope. This is the method used here.

Method

The string was clamped between two pieces of metal to a rigid stand. The length of the pendulum was measured using a tape measure to measure the distance from the clamp to a mark *near the center* of the ball or bob. *(I can't really remember where the mark was so I'm just putting in where I think it was. I did this several years ago.*) The estimated uncertainty being about 0.5mm. The ball was pulled aside to about one twentieth of L* and released. The period was measured with a Pasco timer for 16 cycles and averaged to get an average period. We then plotted T^2 vs L*, calculated the slope and used that to calculate g. When we do this, we assume all of the uncertainties are incorporated into the uncertainty of the slope.

Results and Conclusion

The data is on an attached spreadsheet that shows the raw data and some basic calculations. The results are discussed below.

The slope calculated using a linear regression was 4.0386 s^2/m with a standard error of 0.0088 s^2/m. The percent standard error is 0.22%. The g calculated from this slope is 9.775 m/s^2, and it will also have a percent standard error of 0.22%. (Note that since $g = (4\pi^2)/slope$, the fractional uncertainty of g, f_g, = f_{slope}. In general if $z=x^m$ then $f_z = |m|f_x$. Here m = – 1.) As a result we measured

$$g = 9.775\ m/s^2 \ \pm 0.021\ m/s^2.$$

with this technique. The accepted value is 9.7990m/s^2 in this locale. The difference between our measured value and the accepted value is 0.024m/s^2, just greater than one standard error from our measured value. As a result it is hard to say that there is a statistically significant difference between the two values, and one might say that they are consistent with each other, but not as close as one would hope.

The major problem with this method is that it is difficult to get enough different L*'s because the maximum L is about 2.4m in our rooms. Since we do not want to go much shorter than 1m our range of L*'s is a little too limited and this increases the uncertainty in our slope. It would have been better to be able to do a couple of measurements with longer L's. It would also be better to have more than five points to fit.

Notes

The "Conclusions" sections often benefits from presenting the results in tables, however since we are only comparing two values, our measured one and the accepted value, we don't really need to use a table. If we were comparing several values, we should probably use a table.

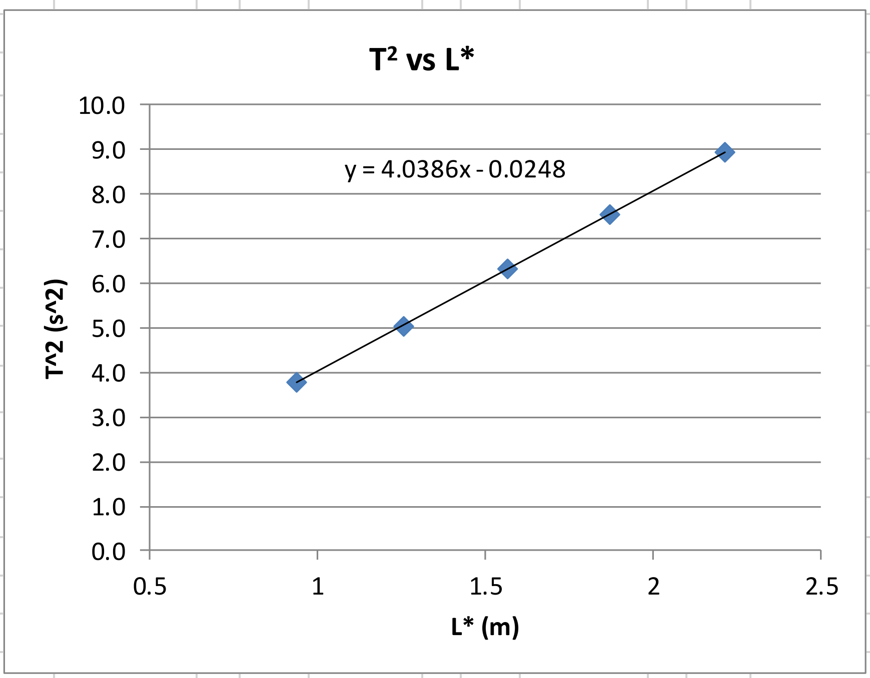
Also the difference between the two values is about 1.14 standard errors. The probability of measuring a value ≥ 1.14 standard errors from the "true mean" is about 0.26 or you would expect this to occur 26% of the time; about a 1 in 4 chance. That is why we can't say the difference is statistically significant. You might <u>suspect</u> that there are some small non-random or systematic errors affecting the measurement, but it is difficult to say that with any certainty.

This is a sample report. The details needed in a report vary with the nature of the experiment, but this should give you an idea what I'm looking for. It is ALWAYS good to ask about what you need to do.

DATA

	Period (s)	Period (s)	Period (s)	Period (s)	Period (s)
1	2.987874	2.745624	2.51107	2.245452	1.944994
2	2.98791	2.744698	2.51129	2.24462	1.945402
3	2.988014	2.74553	2.51128	2.24504	1.945036
4	2.988154	2.745168	2.51139	2.24485	1.945098
5	2.986824	2.745338	2.5114	2.24482	1.94561
6	2.987738	2.744964	2.51163	2.245028	1.944934
7	2.987622	2.745554	2.51132	2.245084	1.94506
8	2.987974	2.745096	2.51095	2.244592	1.945196
9	2.987572	2.745362	2.51132	2.24441	1.945276
10	2.988028	2.745262	2.51098	2.244706	1.945132
11	2.987778	2.744954	2.51167	2.245002	1.944724
12	2.986896	2.745062	2.51059	2.244692	1.945338
13	2.98774	2.745104	2.51108	2.244772	1.945246
14	2.987204	2.74535	2.51128	2.244708	1.944938
15	2.988004	2.745236	2.51111	2.244488	1.945182
16	2.98705	2.745032	2.51085	2.244604	1.94488
Ave=	**2.98765**	**2.74521**	**2.51120**	**2.24480**	**1.94513**
Std=	**0.00043**	**0.00025**	**0.00028**	**0.00026**	**0.00022**
L* =	2.216 m	1.872 m	1.567 m	1.257 m	0.941

L(m)	T(s)	$T^2(s^2)$
0.941	1.9451	3.7835
1.257	2.2448	5.0391
1.567	2.5112	6.3061
1.872	2.7452	7.5362
2.216	2.9876	8.9260



g =	**9.775**	m/s^2
std =	**0.021**	m/s^2
% s=	**0.22%**	
% diff	**-0.24%**	

	Coeff	*Std Error*
Intercept	-0.025	0.014
X Variable	**4.0386**	**0.0088**