# Experimental Errors & Data Analysis

Laboratory investigations involve taking measurements of physical quantities, and the process of taking any measurement always involves some uncertainty or experimental error. In the first experiment, we examine the types of experimental errors and some methods of error and data analysis that will be used in subsequent experiments in the General Physics sequence. The most important sections are 3, 4 and 5.

## 1. Types of errors

Experimental errors can be classified either as **Systematic Errors** or **Random Errors**. Systematic errors are errors associated with particular measurement instruments or techniques that produce consistent errors, e.g. as an improperly calibrated scale might give weights that are always too large. Errors in reading a scale may also produce systematic errors. For example, the position of the mercury meniscus on a thermometer scale determines the temperature (FIG. 1). A person who always reads the thermometer from above (or from below) will introduce a systematic error in his/her data. For the two people below, one would read 21ºC and the other 22ºC. Avoiding systematic errors depends on the skill of the observer to detect and prevent or correct them.

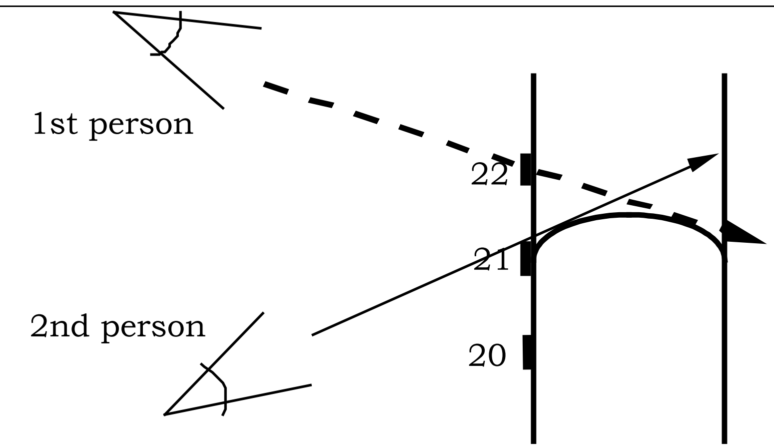



FIGURE 1. Example of personal error in reading a scale due to parallax.

Random errors result from unknown and unpredictable variations in experimental situations. Random errors are just as likely to produce a result that is too large as they are likely to produce one that is too low. Random errors are also referred to as accidental errors and are sometimes beyond the control of the observer. Conditions by which random errors can result include fluctuations in temperature, mechanical vibrations of the experimental setup and estimates of measurement readings by the observer.

One can usually minimize the effect of random errors by making many measurements and averaging the results so that the random fluctuations become statistically insignificant. Much of statistical analysis is concerned with the effects of Random Errors.

## 2. Accuracy and Precision

The accuracy of an experiment is a measure of how close the experimental result comes to the "true value". (This is more complicated when you do not know what the "true value" is.) That is, it is a measure of the correctness of the result. For example, two independent experiments result in the determination of the value of $\pi$ to be 3.140 and 3.141, respectively.

The second result is more accurate because the true value of $\pi$ is 3.142 (to four significant figures).

The precision of an experiment is a measure of how reproducible the result is. That is, it is a measure of the magnitude of the uncertainty of the result due to random errors. For example, two independent experiments give two sets of data with the expressed results and uncertainties of 2.5 ± 0.1 cm and 2.5 ± 0.2 cm, respectively. The first result is more precise than the second, because the uncertainty in the first measurements is between 2.4 and 2.6 cm, whereas the uncertainty in the second measurements is between 2.3 and 2.7 cm.

NOTE: It is possible to be very precise but highly inaccurate. Can you think of such a situation?

The accuracy of an experiment typically depends on both systematic and random errors. The precision of an experiment typically depends on random errors.

## 3. Significant Figures

The degree of certainty of a measurement is implied by the way the result is written or reported. When reading the value of an experimental measurement from a calibrated scale, only a certain number of digits can be obtained or read.

Significant figures are those figures which are known to be reasonably trustworthy; they are figures actually obtained from the measuring instruments, including zeros and estimated figures. The position of the decimal point does not affect the number of significant figures; if a zero is used merely to locate the decimal point, it is not significant, but if it actually represents a value read on the instrument, or estimated, then it is significant. (This is easiest to see if you use scientific notation.)

For example, suppose that a distance is measured with an ordinary centimeter ruler and found to be 52.3 millimeters, where the 3 is estimated; then all three figures are significant. If this number is recorded as 0.0523 meters, then the two zeros are not significant, because they are used merely to locate the decimal point; the number still contains three significant figures. (However, if the estimated figure had been a zero, then the distance should be recorded as 52.0 millimeters, and not 52 millimeters, since the zero would then be a significant figure.) In scientific notation 52.3 mm would be $5.23 \times 10^{-2}$m. Here it is obvious that there are 3 significant figures.

Figures which are not significant should be dropped at the end of a calculation. This not only saves labor, but avoids drawing false conclusions, because the retaining of too many figures implies a greater accuracy greater than one actually has. The following rules may be used for the retention of significant figures in a computation:

a. In addition and subtraction, do not carry the result beyond the first column that contains a doubtful figure. This means that all figures lying to the right of the last column in which all figures are significant should be dropped.

b. In multiplication and division, retain in the result only as many significant figures as the least precise quantity in the data has. (Scientific notation is especially helpful here!)

c. In dropping figures which are not significant, the last figure retained should be unchanged if the first figure dropped is less than 5. It should be increased by 1 if the first figure dropped is greater than 5. If it is 5, round up if it makes the next figure even and don't round if makes the next figure odd.

These rules give the number of significant figures which should appear in the final result, but in computing the intermediate steps, it is useful to carry one more significant figure than is needed in the final result. The following examples illustrate the principles just mentioned:

1. In obtaining the sum of the numbers 806.5, 32.03, 0.0652 and 125.0, they should be written as 806.5, 32.03, 0.07 and 125.0 and added to give 963.60 and then rounded to 963.6. Then the sum is expressed to the correct number of significant figures. Here I kept an "extra" digit during the addition and rounded to the correct number of significant digits at the end.
2. Suppose that a length is measured as 7.62 cm, and a width as 3.81 cm. In these numbers, the third figure has been estimated, and therefore it is a doubtful figure. In computing the area by multiplication, one obtains the result 29.0322 $cm^2$. This number has six significant figures, but neither one of the original quantities is known to this number of figures. There are only three significant figures in each one of the factors. Therefore, only three significant figures should be retained in the result, which should be written as 29.0 $cm^2$.

When your final value is an average of many values, your average, or mean value, may have more significant figures than the individual measurements. **To decide how many significant figures to keep for mean values and standard deviations, read section 5D below.**

## 4. Expressing Experimental Error

### A. Average (Mean) Value

Most experimental measurements are repeated several times, and it is unlikely that identical results will be obtained for all trials. For a set of measurements which are all equally trustworthy or probable and whose differences are due to random errors, the true value is most probably given by the average or mean value. The average or mean value <x> of a set of n measurements is

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad 1$$

### B. Error

The error is defined as the difference between your measured value and the "accepted" value, if there is an accepted value.

$$\text{Error} = (\text{Measured Value}) - (\text{Accepted Value}) \qquad 2$$

### Percent Error

In most experimental work in physics, the percent error (relative error) is of much greater significance than the actual difference between the observed value and the accepted value. The percent error of an experimental value is

$$\text{Percent error} = \left(\frac{\text{Error}}{\text{accepted value}}\right) \times 100\% \qquad 3$$

If the percent error is >0, your measurement is greater than the accepted value. If it is <0, then your measurement is less than the accepted value.

The percent error may not be very useful if the accepted value is zero or too close to zero. Also, some people use the absolute value of the Error in expression 3, so that their percent error is always non-negative.

## C. Percent Difference

Sometimes it is useful to compare equally reliable measurements when there is no generally accepted value. The comparison is expressed as a percent difference, which is the ratio of the absolute difference between the experimental values, $x_2$ and $x_1$, to the average or mean value of the two results expressed as a percent.

$$\text{Percent Difference} = \frac{|x_2 - x_1|}{(x_2 + x_1)/2} \times 100\%$$

4

Dividing by the average or mean value of the experimental values is a logical choice, because there is no way of deciding which of the two results is better.

# 5. Uncertainty

There are several ways of expressing the uncertainty in a measured quantity. The most common way uses the concept of standard deviation which is a measure of how the different measured values vary around the mean value. (Note that the uncertainty is not the same as the error!)

## A. Deviation From the Mean

It is very helpful to know how much the individual measurements differ from the mean. This scatter or dispersion of measurements will give an idea of the precision of the experiment. The deviation, (sometimes referred to as the residual) $d_i$, of any single measurement, $x_i$ , from the mean value, <x>, of the set is

$$d_i = x_i - \langle x \rangle \qquad 5$$

The deviation may be positive or negative, since some measurements are larger than the mean and some are smaller. The average of the deviation of a set of measurements is always zero. (This is a result of the definition of the mean value. We choose the mean value to make the sum of deviations zero.)

## B. Standard Deviation: σ

Note: There is a subtle difference between the standard deviation of a total population and the estimate of the standard deviation based on a sample of that population. The formulas here for the standard deviation are those for the estimate of the standard deviation based on a sample of the population. **We will not worry about this distinction in this class**. It is usually covered in more advanced classes or in statistics classes.

It is mathematically convenient to work with the squares of the deviations. The standard deviation, σ, is defined by

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} d_i^2 \qquad 6$$

The square of σ is called the variance of a set of measurements, and the standard deviation is sometimes called the root-mean-square (RMS) deviation. The standard deviation represents the average of the square of the variation of individual measurements about the mean value. The above definitions will be illustrated with the following set of numbers that represent the measurement of the diameter of a disk in cm, 5.42, 6.18, 5.70, 6.01, and 6.32. Then the mean value of this set is

<x> = (5.42 + 6.18 + 5.70 + 6.01 + 6.32)/5 = 5.93cm

The deviations di are

$d_1 = 5.42 - 5.93 = - 0.51cm$
$d_2 = 6.18 - 5.93 = + 0.25cm$
$d_3 = 5.70 - 5.93 = - 0.23cm$
$d_4 = 6.01 - 5.93 = + 0.08cm$
$d_5 = 6.32 - 5.93 = + 0.39cm$

The standard deviation σ is

$$\sigma = \sqrt{(0.26 + 0.06 + 0.05 + 0.01 + 0.15)cm^2 / 4} = 0.37cm$$

If the errors are random (and normally distributed) then about 2/3 of the measurements should be within ± σ of the mean value, or between 5.56cm and 6.30cm. About 95% of the individual measurements should be within ± 2σ of the mean value, or between 5.19cm and 6.67cm. σ is an estimate of the uncertainty of a **Single Measurement** due to random errors.

What is the **uncertainty of the MEAN VALUE** for a set of measurements? It turns out that our best estimate of the uncertainty of the mean of a set of n measurements, $\sigma_n$, is the uncertainty for a single measurement divided by the square root of n, or

$$\sigma_n = \frac{\sigma}{\sqrt{n}}. \quad 7$$

This $\sigma_n$ is the standard deviation for the MEAN VALUE of a set of n measurements. For the above example, $\sigma_n = 0.37cm/\sqrt{5} = 0.17cm$. I would represent the diameter of the disk as the mean value ± $\sigma_n$, or

diameter = 5.93 cm ± 0.17 cm.

Note that as n increases, $\sigma_n$ becomes smaller. Making many measurements can give you a more accurate estimate of the true diameter, **IF** the fluctuations are due to random errors. (This can be a **BIG IF**.) This result is very important in more advanced applications.

If your measured value is within ± $\sigma_n$ of the predicted value, there is little statistically significant difference between the two values. They appear to be about the same. You will often be making comparisons like this in your labs. (Note that some people use ± $2\sigma_n$ for the comparison instead of ± $\sigma_n$.)

Also note that the standard deviation may have units. The standard deviation will have the same units as the value it is referring to.

> The standard deviation is related to the squares of the deviations or distances of individual measurements from the mean value, m. Instead of looking at deviations from the mean, we could look at deviations from an arbitrary number, z. Then the deviation of $x_i$ from z is $d_i = (x_i - z)$. If one looks at the sum of the $d_i$ squared,
>
> $$s^2 = \sum_{i=1}^{n} (x_i - z)^2$$
>
> and choose the z that minimizes the quantity $s^2$, that z turns out to be the mean value as defined above. The mean value produces the "minimum" standard deviation. Choosing parameters (in this case the mean) to minimize the sum of the squares of the deviations is given the name "least squares". Least squares principles are often used to define the "best value or best estimate" of a quantity, i.e. they are best because they minimize the sum of the $(d_i)^2$.

## C. Fractional Standard Deviation

Sometimes it is useful to use the fractional standard deviation. If $\sigma_d$ is the standard deviation for the diameter and <d> is the mean value of d, the fractional standard deviation, $f_d$, is

$$f_d = \frac{\sigma_d}{< d >} \quad 8$$

The fractional standard deviation is just a number, it does not have any units, and it is often quoted as a per cent. For instance, if the mean diameter is 5.93cm and $\sigma_d$ = 0.37cm, then the fractional standard deviation is $f_d$ = 0.37cm/5.93cm = 0.062 = 6.2%. The fractional standard deviation for the mean value of d is

$$f_d = \frac{\sigma_{<d>}}{< d >} = \frac{0.17cm}{5.93cm} = 0.028 = 2.8\% \quad 9$$

## D. Standard Deviation and Significant figures

**If you make a set of measurements and calculate a mean value of 6.238 and a standard deviation of 0.1569, you should report your results as**

**6.24 ± 0.16**

**You should not keep more than two significant figures in the standard deviation or uncertainty. In your reporting of the mean value you should not report digits past the uncertainty. Since the last figure in the standard deviation above is in the hundredths digit, round the mean value to that digit. If the most significant digit in the standard deviation is greater than 5, e.g. if the standard deviation for the above measurement was 0.087, some might just keep one digit in the standard deviation and report it as**

**6.24 ± 0.09**

I usually keep two significant figures in the standard deviation and this seems to be the standard. ***I recommend keeping two significant figures***.

There is another consideration. If your measurement resolution is low, that can make your standard deviation look artificially low. If you only have one digit of resolution, you might get the same value each time. That will make your standard deviation zero, but you really don't have a good idea of what it is, and should probably give the result to one digit.

## 6. Propagation of Errors

If I want to estimate the value of π by measuring the circumference (C) and diameter (d) of a circle, I ***calculate*** π by

$$\pi = \frac{C}{d} \quad 10.$$

If I have made many measurements of d and C and have mean values, <d> = 10.00 cm and <C> = 31.10cm, and standard deviations of the mean, $\sigma_d$ = 0.20 cm and $\sigma_C$ = 0.30cm, for both, what is my best estimate of π and $\sigma_\pi$? The best estimate of π is to use the mean values of d and C , <d> and <C>, in eqn. 1, $\pi = \frac{< C >}{< d >} = 3.11$. <u>Note that I have not measured π directly, but rather calculated it from other measured quantities</u>.

Estimating $\sigma_\pi$ from $\sigma_d$ and $\sigma_C$ is a little more complicated; so I'll just give you the rules here and discuss it in more detail in Appendix A. It is often useful to use the fractional standard deviation in the propagation of errors. If you do this the fractional standard deviation for π is

$$f_\pi^2 = f_d^2 + f_C^2 \quad 11.$$

which is a relatively simple result. The square of the fractional standard deviation of pi is equal to the sum of the squares of the fractional standard deviations of the diameter and the circumference. (Equation 11 is an approximation, but it is good enough for this course.)

Using eqn. 11 and that $f_d = 0.020$ and $f_C = 0.0096$, one gets that $(f_p)^2 = 0.00049$ and $f_p = 0.022$ or that $\sigma_\pi = 0.022 \times 3.11 = 0.068$. In this case $\pi$ and $\sigma_\pi$ are dimensionless. Most of our mean values and their standard deviations will have dimensions. You need to include them!

There are a number of simple expressions similar to equation. 11 that are derived from equation A2 below. I have given two of them below. Number one below is often useful. Fractional standard deviations are most useful when the relation is z equals a product or ratio of powers of x and y. If z = f(x,y) and a, b, n and m are constants, then

| | Functional relationship between z, x, & y | Relationship for $\sigma_z$, $\sigma_x$ & $\sigma_y$ or $f_z$, $f_x$ & $f_y$. |
|---|---|---|
| 1 | $z = ax^n y^m$ | $f_z^2 = (nf_x)^2 + (mf_y)^2$ |
| 2 | $z = ax + by$ | $\sigma_z^2 = (a\sigma_x)^2 + (b\sigma_y)^2$ |

Consider the following example. You calculate a velocity by measuring the time t it takes an object to go a distance d; v = d/t. Then you use this velocity to calculate a kinetic energy. What is the standard deviation in the KE due to the standard deviations in t and d? Assume $\langle d \rangle = 0.0620\text{m}$ with $\sigma_d = 0.0050\text{m}$ and $t = 0.0440\text{s}$ with $\sigma_t = 0.0020\text{s}$. First, the KE is given by

$$KE = \frac{mv^2}{2} = \frac{m}{2}\left(\frac{d}{t}\right)^2 \qquad 12$$

This is like row 1 in the table above with a = m/2, x = d, y = t with n=2 and m = − 2. The fractional standard deviations for d and t are $f_d = 0.081$ and $f_t = 0.045$, so

$$f_{KE}^2 = (2 \times 0.081)^2 + (-2 \times 0.045)^2 = 0.034 \qquad 13$$

as a result $f_{KE} = (0.034)^{1/2} = 0.18$. The fractional standard deviation for the KE is 18%. Most of this comes from the 5 mm standard deviation for the distance d.

## 7. Estimation of Uncertainties

Sometimes you are not able to calculate a standard deviation for a quantity. Consider a pendulum where you measure the length, L, of the pendulum and its period, which is the time for one complete cycle, to find g, the local gravitational field. You would measure the length and the time for 10 cycles of the pendulum, $t_{10}$. The period is $T = t_{10}/10$. You may only have one measurement of the each quantity. You may have to estimate what your uncertainty in both the time measurement and length measurement are. If you are measuring the distance with a ruler, you might estimate that you can measure the distances to the nearest millimeter so the uncertainty is ± 0.5mm. Then this would be your uncertainty and your estimate of the standard deviation of the length would be ESTIMATED as 0.5mm. Similarly for the period, you may have to estimate how accurately you can use a stopwatch to measure the period. It will depend on your judgment and reflexes. You might estimate that you can use a stopwatch to measure time intervals to the nearest 0.5s, or that you have an uncertainty of ± 0.25s. However this is the uncertainty in $t_{10}$. Your period is one tenth of $t_{10}$, so your estimated uncertainty in T is ± 0.25s/10 = ± 0.025s. Then you would use $\sigma_L = 0.50\text{mm}$ and $\sigma_T = 0.025\text{s}$, but these are estimated rather than the outcome of repeated measurements.

You could repeatedly measure the length, but since you know what you measured the first time, you might be biased and measure the same value again and again, i.e. producing a systematic error. This would produce an unrealistically low estimate of the uncertainty in the length. It would be better if a number of different people made the measurement without knowing each other's readings and then pool all their results to find a mean value for the length and a standard deviation for the length. If you just have your own measurement, it might be better to estimate how well you can make the measurement and use that as your uncertainty.

## APPENDIX A: A Note on Propagation of Errors

Many times we do not measure the quantity we want to know directly. If I want the resistance of a resistor, I don't actually measure the resistance directly, but rather measure the current through the resistor, I, and the voltage across the resistor, V, and ***CALCULATE*** the resistance as R = V/I. If I want to estimate the value of π, I would measure the circumference, C, and diameter, d, of a circle, and ***calculate*** π by

$$\pi = C/d \qquad \text{A1.}$$

If I have made many measurements of d and C and have mean values, <d> = 10.00 cm and <C> = 31.10cm, and standard deviations of the **mean**, $\sigma_d$ = 0.20 cm and $\sigma_C$ = 0.30cm, for both, what is my best estimate of π and $\sigma_\pi$? The best estimate of π is to use <d> and <C> in 1, or π = <C>/<d>= 3.110.

Estimating $\sigma_\pi$ from $\sigma_d$ and $\sigma_C$ is a little more complicated; so I'll just give you the rules without the justification. If I have a quantity g as a function of x and y, g = f(x,y), then my best estimate of g is g = f(<x>,<y>) and

$$\sigma_g^2 = \sigma_x^2 \left( \frac{\partial f}{\partial x} \right)^2 + \sigma_y^2 \left( \frac{\partial f}{\partial y} \right)^2 \qquad \text{A2}$$

where the partial derivatives are to be evaluated at the mean values of x and y. This is an approximation that works if f(x,y) does not vary too rapidly about the point (<x>,<y>) and the fractional standard deviations are not too large.

If we apply this to the problem above, π = f(d,C) = C/d then

(Mf/Md) = $-$ C/d$^2$

(Mf/MC) = 1/d

Inserting these into B2 yields

$$(\sigma_\pi)^2 = (\sigma_d)^2(-C/d^2)^2 + (\sigma_C)^2(1/d)^2$$

or

$$(\sigma_\pi)^2 = (0.2\text{cm})^2(-0.31/\text{cm})^2 + (0.3\text{cm})^2(0.1/\text{cm})^2.$$

Calculating $\sigma_\pi$ yields a value of 0.069. As a result our best estimate of pi is π = 3.110 ± 0.069. Note that in this case, pi and $\sigma_\pi$ are dimensionless. In many cases means and standard deviations will have dimensions. **Be sure you indicate the dimensions, if there are any**.

## APPENDIX B: GRAPHS and LINEAR REGREESSION

### A. Graphs

It is often convenient to represent experimental data in graphical form, not only for reporting, but also to obtain information. Quantities are commonly plotted using rectangular Cartesian axes (X and Y). The horizontal axis (X) is called the abscissa, and the vertical axis (Y), the ordinate. The location of a point on the graph is defined by its coordinates x and y, written (x,y).

When plotting data, choose scales so that most of the graph paper is used. When the data points are plotted, one usually, but not always, draws a smooth line connecting the points. "Smooth" suggests that the line does not have to pass exactly through each point, but connects the general areas of significance as indicated by the circles around the data points. (If you are going to do a linear regression on the data points, it looks better if you do not draw a smooth line connecting the points.)

In the case where several determinations of each experimental quantity are made, the standard deviation may be plotted as error bars. For example, the following data might represent the measured period for the simple harmonic motion of a mass on a spring of negligible mass. The period is measured for different masses and the period vs. mass is plotted in Figure A. (These are not real measurements; they are just made up as an illustration.) A smooth line is drawn within the error bars.

| | Mass (kg) | Period (S) |
|---|---|---|
| 1 | 0.1 | 0.63 |
| 2 | 0.5 | 1.4 |
| 3 | 1 | 1.99 |
| 4 | 2 | 2.81 |
| 5 | 4 | 3.97 |
| 6 | 6 | 4.87 |
| 7 | 7.5 | 5.44 |
| 8 | 9 | 5.96 |

Note that the plot is not linear, so the curve through the points is not a straight line. For the simple harmonic motion of a mass M suspended from a spring of spring constant k, the period, T, *should* be proportional to (mass)$^{½}$.

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad \text{B 1.}$$

(In this data set we have assumed that the mass is very well known, so that most of the uncertainty is in the measurement of the period.)

***A useful feature of graphs is that you can often tell a great deal about the relationship between two variables just by looking at the plot***. From the graph above it is obvious that the period is not a linear function of mass. However, it is also apparent that the functional relationship is regular and smooth.

If one suspected that $T^2$ is proportional to the mass, then one could plot $T^2$ vs. mass instead of T vs. mass. This is done in the graph at the right. ***It is a straight line, indicating $T^2$ is linearly proportional to the mass***. (T^2 means period squared. On the scale of this plot, the error bars are the size of the data points, so they do not show.) The slope of the line gives the constant of proportionality between $T^2$ and the mass

***Graphs should have:***

- ***a. Each axis labeled with the quantity plotted.***
- ***b. The units of the quantities plotted.***
- ***c. The title of the graph. (The title should be descriptive, not just "Graph 1".)***

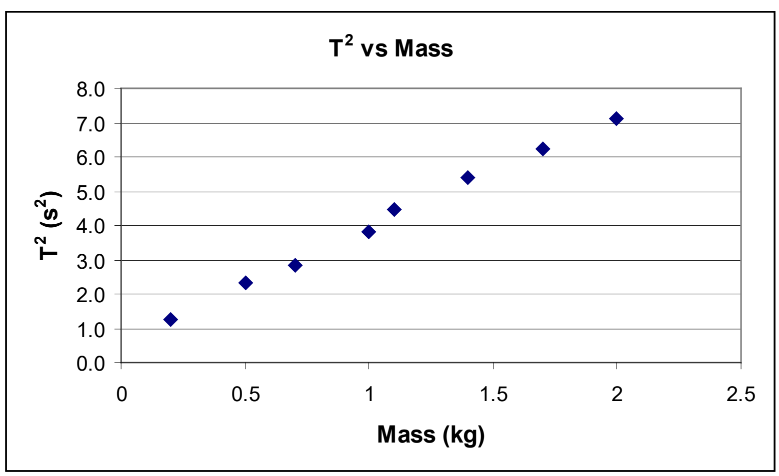
## B. Linear Regression

If the spring's mass is **NOT** negligible, we might suspect that mass and $T^2$ are linearly related to each other, or that

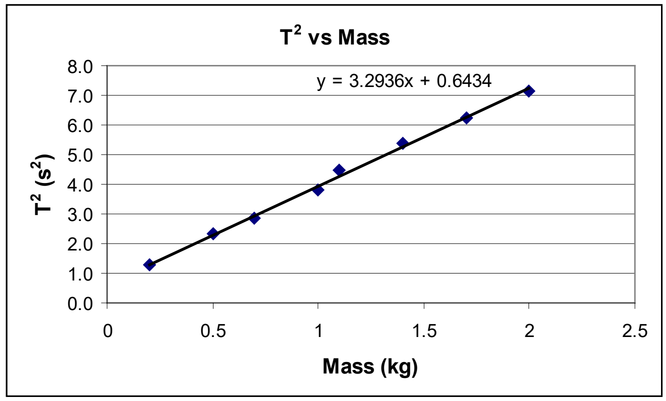
$$T^2 = a + bM \qquad \text{B 2.}$$

where a and b are constants, then a plot of $T^2$ vs M should be a straight line. The slope is b and a is the intercept. However if there are measurements errors, then the points may not all lay on a straight line. The data might look like the data at below with the plot as shown.

| M (kg) | T (s) | $T^2$ ($s^2$) |
|---|---|---|
| 0.2 | 1.13 | 1.27 |
| 0.5 | 1.53 | 2.33 |
| 0.7 | 1.69 | 2.84 |
| 1 | 1.95 | 3.81 |
| 1.1 | 2.12 | 4.48 |
| 1.4 | 2.32 | 5.39 |
| 1.7 | 2.49 | 6.22 |
| 2 | 2.67 | 7.13 |



These points do not all lay on a straight line. There is a way to estimate the "best" straight line that would fit these points. It is called a linear regression. (This estimate will minimize the squares of the deviations of the points from the line, so it is also called a least squares fit.) The regression routine in EXCEL can calculate the slope, b, and the intercept, a, and give the uncertainty in the slope and intercept. (If you ask the graphing routine to do the calculation, it can display the slope and intercept on the chart, but it will not give you the uncertainties.) The regression line for the above data points is shown at the right. Note that it also gives the slope and intercept.




To get the uncertainties in the slope and intercept you have to go to the DATA ANALYSIS section of EXCEL and under the TOOLS look for the linear regression routine. If you run that, the output looks like the one below. The information you will need is highlighted in boldface. (I've done the highlighting, EXCEL will not highlight it.) The intercept is in the column marked COEFFICIENTS and is 0.64342; and the slope is the X VARIABLE 1 in the COEFFICIENTS column and is 3.29359. The uncertainty in each of these is in the column labeled STANDARD ERROR, which is an estimate of the standard deviation. For the intercept it is 0.09988 and for the slope it is 0.08210. They contain a "lot of digits" so I would round them and say the intercept = 0.64 $s^2$ ± 0.10 $s^2$ and the slope = 3.294 $s^2$/kg ± 0.082 $s^2$/kg. These both have units. The units of the intercept are $s^2$ and for the slope, $s^2$/kg, so I have included them. You should include the units! (The columns of UPPER 95% and LOWER 95% are more sophisticated ways of estimating the uncertainty in the slope and intercept.)

SUMMARY OUTPUT

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.99814 |
| R Square | 0.99629 |
| Adjusted R Square | 0.99567 |
| Standard Error | 0.13226 |
| Observations | 8 |

ANOVA

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 28.14982 | 28.14982 | 1609.34 | 0.00000 |
| Residual | 6 | 0.10495 | 0.01749 | | |
| Total | 7 | 28.25477 | | | |

| | ***Coefficients*** | ***Standard Error*** | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0%* | *Upper 95.0%* |
|---|---|---|---|---|---|---|---|---|
| **Intercept** | **0.64342** | **0.09988** | 6.44191 | 0.00066 | 0.39902 | 0.88781 | 0.39902 | 0.88781 |
| **X Variable 1** | **3.29359** | **0.08210** | 40.11654 | 0.00000 | 3.09269 | 3.49448 | 3.09269 | 3.49448 |