Radioactivity

Introduction

Radioactivity is the spontaneous emission of some types of radiation by atomic nuclei. Not all nuclei are capable of emitting radiation, those that do not (such as $^{12}$C) are termed stable nuclei. Those that do are termed radioactive nuclei or radioisotopes.

Discovery of radioactivity

- Wilhelm Conrad Roentgen (1845-1923) was the discoverer of X rays. In 1895 he was working on an experiment on vacuum tubes when he saw fluorescent screen on a nearby table start to glow. Roentgen determined that the fluorescence was caused by invisible rays originating from the vacuum tube. Unlike other rays known at the time (such as light) these mysterious rays could penetrate the opaque black paper wrapped around the tube, as well as other materials. Roentgen named them X rays.

We now know that X rays do not originate from the nucleus, but from electrons. Strictly speaking they are not part of nuclear physics, or radioactivity. However, they do start our discussion of the discovery of radioactivity, and we will return to them in a later discussion of the hazards of ionizing radiation, which includes both radioactive particles and X rays.

- The discovery of radioactivity is credited to Henri Becquerel (1852-1908). He knew of Roentgen’s discovered of X rays from the fluorescence they produced, and possessed a ready source of fluorescent materials with which to pursue his own investigations of these mysterious rays.

In 1896 Becquerel was working with potassium uranyl sulfate, which he exposed to sunlight and then used to produce images on photographic plates wrapped in black paper. Becquerel concluded "that the phosphorescent substance in question emits radiation which penetrates paper opaque to light." Initially he believed that the sunlight was being absorbed by the uranium which then emitted X rays. However, he later found that he could repeat the experiment even when the skies were cloudy. This meant that the uranium emitted radiation even by itself, without the benefit of being “activated”. Becquerel had discovered radioactivity, the spontaneous emission of radiation by a material.

Later, Becquerel demonstrated that the radiation emitted by uranium shared certain characteristics with X rays but, unlike X rays, could be deflected by a magnetic field and
therefore must consist of charged particles. For his discovery of radioactivity, Becquerel was awarded the 1903 Nobel Prize for physics. One of the basic units of radioactivity (the Becquerel) is named after him (see below).

- Marie Curie (1867-1934) and Pierre Curie (1859-1906) began investigating the phenomenon of radioactivity which had been recently discovered by Becquerel. They managed to discover the radioactive elements polonium and radium, and were the first to isolate a radioactive element (radium). Even the term “radioactivity” was coined by Marie Curie. The other basic unit of radioactivity (the Curie) is named after them.

**Conservation Laws**

The radioactive decay of unstable nuclei is subject to the same conservation laws that are applicable to other areas of physics

- Electrical charge, or equivalently atomic number. The total charge, given by the subscripts on the isotopic nuclei description, must be the same before and after the radioactive decay. However, this does not mean that the total number of protons remains constant when electrons or positrons are produced (see β decay below).

- Number of nucleons, given by the superscripts on the isotopic nuclei description. This is almost equivalent to conservation of mass, except that the mass of a given nucleus is not quite an integer. In fact the total mass always decreases very slightly (see below).

- Energy, including the energy equivalent of mass.

- Momentum. Conservation of momentum is a factor in determining the directions at which emitted particles are emitted, but we will not be pursuing this aspect further.

**Types of radioactive decay**

*Alpha (α) Decay*

In the α decay process the nucleus of the atom emits a particle which consists of two protons and two neutrons, known as an α particle. Later on, after the α particle picks up two neutrons it will become a normal helium atom, which means that an α particle is also equivalent to the nucleus of a helium atom, ⁴He.
$^{235}\text{U}$ is an example of a nucleus which will decay by emission of an $\alpha$ particle. Since the $\alpha$ particle takes away two protons, conservation of charge dictates that the daughter nucleus left behind must have two fewer charges (protons) than the parent nucleus. Since the original uranium atom has an atomic number of 92, then the daughter nucleus must have an atomic number of 90, which corresponds to the element thorium. Conservation of nucleon number means that the daughter must also have four fewer nucleons. That means that its atomic mass must be 231. The daughter isotope is therefore $^{231}\text{Th}$.

$$^{235}_{92}\text{U} \rightarrow ^{231}_{90}\text{Th} + ^{4}_{2}\text{He}$$

The $\alpha$ particle which is emitted has a large kinetic energy, meaning that it is moving with a speed which is an appreciable fraction of the speed of light. (There is also a smaller amount of energy which is the kinetic energy of the thorium daughter nucleus.) We can therefore modify the above reaction equation to read

$$^{235}_{92}\text{U} \rightarrow ^{231}_{90}\text{Th} + ^{4}_{2}\text{He} + \text{energy}$$

in agreement with conservation of energy. The amount of energy produced is equivalent to the amount of mass that is lost. We shall look at this in more detail later.

**Beta ($\beta$) decay**

There are two variations of $\beta$ decay, one in which a negatively charged particle is emitted, the other in which a positively charged particle is emitted. To distinguish between the two they are termed $\beta^-$ and $\beta^+$ decay.

$\beta^-$ decay

In $\beta^-$ decay the nucleus ejects a high speed electron. Since an electron is not a nucleon the number of nucleons in the daughter nucleus is the same as that of the parent nucleus.
However, since the change of the electron is -1, then the charge of the daughter must be one more than that of the parent.

An example of a beta emitter is $^{137}$Cs. Cesium has an atomic number of 55, which means that the daughter must have an atomic number of 56, corresponding to barium. The beta decay of $^{137}$Cs is therefore

$$^{55}_{\text{Cs}} \rightarrow ^{137}_{56}\text{Ba} + ^0_{-1}\text{e}^- + \text{energy}$$

As was the case for $\alpha$ decay, the amount of energy produced is equivalent to the amount of mass that is lost. We shall look at this in more detail later.

$\beta^+$ decay

In $\beta^+$ decay the nucleus ejects a particle known as a positron. A positron is the antiparticle to the electron, it has the same mass, but instead of being negatively charged it is positively charged. Since the position, like the electron, is not a nucleon, the number of nucleons in the daughter nucleus is the same as that of the parent nucleus. However, since the change of the electron is +1, then the charge of the daughter must be one less than that of the parent.

An example of a beta emitter is $^{18}$F. Fluorine has an atomic number of 9, which means that the daughter must have an atomic number of 8, corresponding to oxygen. The $\beta^+$ decay of $^{18}$F is therefore

$$^{18}_{8}\text{F} \rightarrow ^{18}_{8}\text{O} + ^0_{+1}\text{e}^+ + \text{energy}$$

Positron emitters are used in medicine in the diagnostic technique known as Positron Emission Tomography (PET scans, for short).
Energy production in beta decay

Measurements of the energy of the helium nucleus (α particle) following the α decay of a given alpha emitter always yield the same result. However, the energies of the electrons (or positrons) following β decay have a range of values, up to and including a maximum value. Since the Principle of Conservation of Energy is held to be inviolate it was postulated that there is a second, and unseen, particle emitted as the result of β decay, and that the energy is shared in a varying portion between the electron (or positron) and this unseen particle. This unknown particle was termed a neutrino, with no charge or mass, and an interaction with matter which is so weak that its existence was not verified experimentally for many years after it was first postulated to exist.

The equations for β decay ought to be modified to include the neutrino. If we include the neutrino in the β decay of $^{137}\text{Cs}$ we get

$$^{137}_{55}\text{Cs} \rightarrow ^{137}_{56}\text{Ba} + ^0_{-1}\text{e}^- + ^0_0\nu + \text{energy}$$

Gamma (γ) decay

Gamma decay is fundamentally different to both the α and β decay processes in that the parent nucleus does not eject a particle. As a result the daughter nucleus is the same as the parent nucleus. However, a γ ray is produced, a small packet (termed a photon) of energy which is fundamentally identical to light, except that its energy is orders of magnitude higher.

Gamma decay follows β decay. The daughter nucleus of the β decay process is produced in an excited state, termed a metastable state. In this state the nucleons of the daughter nucleus have an excess of energy which they can release as a γ ray. For example, in the example of β decay of $^{137}\text{Cs}$ (see above), the $^{137}\text{Ba}$ nucleus is produced in a metastable state. At some time later the excited barium nucleus releases this energy in the form of a γ ray

$$^{137}_{55}\text{Cs} \rightarrow ^{137m}_{56}\text{Ba} + ^0_{-1}\text{e}^- + ^0_0\nu + \text{energy}$$

$$^{137m}_{55}\text{Ba} \rightarrow ^{137}_{56}\text{Ba} + ^0_0\gamma$$

Note that in this pair of equations the intermediate excited state of the barium nucleus is designated with a superscript m alongside its atomic mass. Note also that the gamma ray, since it is not a particle, has neither mass nor charge.
Radioactive series

So far we have looked at radioactivity as a single step process. A parent nucleus undergoes a radioactive decay and produces a daughter nucleus. However the daughter nucleus might also be radioactive, producing another daughter nucleus. For example the $\alpha$ decay of $^{238}\text{U}$ produces $^{234}\text{Th}$ as the daughter nucleus and $^{234}\text{Th}$ is itself a $\beta$ emitter, producing $^{234}\text{Pa}$, which itself a $\beta$ emitter, and so on. There is a succession of radioactive isotopes, each one being the daughter of the previous parent isotope, until a stable isotope is formed as the daughter of the last step in the series, at which point the series stops.

When a nucleus decays by the emission of an $\alpha$ particle then its atomic number of the daughter is four less than that of the parent. Following either $\beta$ or $\gamma$ decay the atomic number does not change at all between parent and daughter. It follows that if we start with $^{238}\text{U}$ then we can only produce isotopes with atomic numbers 238, 234, 230, etc. The isotopes with atomic numbers not in this sequence must start with a different initial isotope. To cover all possible atomic numbers there has to be four series

1. The 4n series, which starts with $^{232}\text{Th}$ and ends with the stable isotope $^{208}\text{Pb}$
2. The 4n+1 series, which starts with $^{237}\text{Np}$ and ends with $^{209}\text{Bi}$
3. The 4n+2 series, which starts with $^{238}\text{U}$ and ends with $^{206}\text{Pb}$
4. The 4n+3 series, which starts with $^{235}\text{U}$ and ends with $^{207}\text{Pb}$

The atomic numbers in each series can be found by substituting integer values of the variable n.

Further details of each series can be found online.

Half Lives

Although we might know that a particular nucleus is radioactive, and that it will at some time produce an energetic particle or ray, we cannot tell when it will do so. An individual nucleus might undergo radioactive decay in the next few seconds, or it might not do so for millions of years. We cannot say for certain, the actual time is random. The best that we can do is to assign a probability of the nucleus decaying per second.

However, when it comes to a large number of nuclei (and even a small sample of a radioactive material contains a very large number of nuclei) we can use the techniques of statistics to draw some overall conclusions.
For a large number of nuclei we can write for the activity of the sample

\[ \text{Activity} = \text{Number of decays per second} = \text{constant} \times \text{number of parent nuclei} \]

The constant in this equation is referred to as the *decay constant*, and is given the symbol \( k \). This equation can then be written symbolically as

\[ A = k N \]

The decay constant is a fixed number for any given nucleus, and we have never found any way to change it. However, if we compare different nuclei then the value of the decay constant is highly variable. For a highly radioactive isotope the decay constant is large, for one which is only weakly radioactive the decay constant is a small number.

Activity is measured in Becquerel (Bq) where 1 Bq = 1 disintegration per second. Also used is the Curie (Ci) where 1 Ci = 3.7 x \( 10^{10} \) Bq = 3.7 x \( 10^{10} \) disintegrations per second.

The activity of the sample is also the rate at which parent nuclei are being lost, one \(^{235}\text{U}\) nucleus will produce one \((\text{and only one})\) alpha particle. We can therefore also write

Change in number of parent nuclei per second = - constant \times number of parent nuclei

which can be written symbolically as

\[ \frac{\Delta N}{\Delta t} = -k N \]

where \( \Delta N \) is the change on the number of parent nuclei is the time interval \( \Delta t \), and the extra '-' sign on the right hand side of the equation indicates that the number of parent nuclei is decreasing. This equation is easily solved (if you know calculus) for the number of parent nuclei remaining at time \( t \) to give

\[ N = N_0 e^{-kt} \]

In this equation \( N \) is the number of remaining parent nuclei, and \( N_0 \) is its initial value (at \( t=0 \)). The dependence of \( N \) as a function of time is shown in figure 3. The shape is known as an exponential decay. One property of the exponential decay is that starting at any point on the graph, then

![Radioactive Decay](image-url)

_Fig 3: Typical decay curve for a radioactive isotope_
increasing the time by a fixed number the number of remaining parent nuclei decreases to \( \frac{1}{2} \) of its initial value. For this reason the increase in time is known as the \textit{half life} of the isotope. It is related to the decay constant by

\[
T_{1/2} = \frac{\ln(2)}{k}
\]

Since the decay constant is a fixed number for a given isotope then the half life is also fixed, no process has yet been found to change it. However, as with the decay constant, the half life varies widely from isotope to isotope. For some it is a fraction of a second, for others the half life can be billions of years.

Since the activity (\( A \)) is directly related to the number of remaining parent nuclei the activity of a sample also shows the same exponential decay

\[
A = A_o e^{-kt}
\]

In figure 3, the colored line is the best fit to the data using this equation. From it an experimentalist can easily determine the value of the decay constant \( k \), and hence also the half life of the isotope.

**Measurement of half life**

For some isotopes, such as the example of \(^{137}\text{m}\text{Ba}\) shown in figure 3, the measurement of the half life is straightforward. One prepares a sample of the isotope, free from any other radioactive species, and then measures the activity of the sample at different times. The rate at which the activity decreases immediately gives the half life.

There are some limitations to this direct measurement of the half life

- It must be possible to isolate the desired isotope from other radioactive isotopes.
- The half life must not be too short, or it will have decayed away before sufficient number of counts have been recorded.
- The half life must not be too long, or no appreciable change in the number of counts will be recorded in any reasonable time for making the measurement. For example, how would you measure the half life of \(^{238}\text{U}\) (about 4.5 billion years) when the change in the activity is only about \( 10^{-6}\% \) over the roughly 75 years lifespan of a human being?
Even so, there are a large number of isotopes whose half life can be measured in this way, with half lives ranging from a few seconds to tens, even hundreds, of years.

For isotopes with half lives outside of this range we have to use a different method. Suppose I have a sample which contains two radioactive isotopes, a parent isotope A and a daughter isotope B, and that the sample is old enough that the rate of production of the daughter from the parent A is equal to the rate of decay to the daughter isotope B to a third isotope C. From the definition of activity we can write for the daughter isotope in which the subscripts refer to the parent and daughter nuclei. The first term on the right hand side is the rate at which daughter nuclei are being produced (equal to the rate at which parent nuclei are being lost) and the second term is the rate at which daughter nuclei are being lost. This can be simplified to give

\[
\frac{\Delta N_B}{\Delta t} = k_A N_A - k_B N_B = 0
\]

Now, suppose that either one of these isotopes (for example the daughter) has a known half life. If we now measure the number of nuclei of parent and daughter, then this equation can be rearranged to give the half life of the other (in our example the parent).

This idea can be extended to a sample containing multiple radiative isotopes, such a naturally occurring ore with one or more entire radioactive series (see above). Such ore sample are as old as the Earth (or nearly so) and long since managed to establish the condition that the rate of production of any one daughter is matched by its own rate of decay. The whole sample gradually becomes less radioactive with a half life equal to that of the first and longest lived isotope in the series, but as it does so the relative amounts of the different isotopes remain constant.

We can then write for the whole series

\[
k_A N_A = k_B N_B = k_C N_C = k_D N_D = k_E N_E = k_F N_F = \ldots.
\]

until the stable last isotope in the series is encountered. If follows that if only one half life is known from direct measurement, then all the others can be determined if the relative amounts of the other isotopes are measured.
**Calculations involving half life**

From the relationship between the activity and time we can calculate the activity of a given sample at any later time. The same relationship can also be rearranged to solve for the time when the activity is \( A \). Rearrangement of the equation gives

\[
t = (1/k) \ln(N_0/N) = (1/k) \ln(A_0/A)
\]

Equations * and * form a pair which allow us to calculate the activity if the time is known, or to calculate the time if the activity is known.

**Example 1:** What fraction of a 10 gram sample of remains after one half life has passed? How much after 4 half lives have passed?

1. By definition, after one half life one half of the original sample remains, that is 5 grams
2. After 4 half lives the fraction remaining is \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} \) of the original sample, or \( \frac{1}{16} \) of 10 grams = 0.625 grams.

**Example 2:** The activity of a sample of \( ^{184}\text{Hg} \) is 750 Bq. What will be its activity after 61.8 seconds?

\( ^{184}\text{Hg} \) is an alpha emitter with a half life of 30.9 seconds. This is exactly two half lives, and the activity will have decreased by a factor of \( \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \). The activity is therefore \( \frac{1}{4} \times 750 = 188 \) Bq.

Note that in these two examples the elapsed time is an integer number of half lives. In that case we need only multiply the initial amount of the sample (or the initial activity) by the same number of factors of \( \frac{1}{2} \). The next example is a little harder.

**Example 3:** The activity of a sample of \( ^{184}\text{Hg} \) is 750 Bq. What will be its activity after 80 seconds?

In this example the elapsed time is not an integer number of half lives \( (80/30.9 = 2.6) \). In this case we need to rely on the formula above.

1. As in example 2, the half life is 30.9 seconds. From this figure we can calculate the decay constant, \( k = \ln(2)/T_{1/2} = \ln(2) / 30.9 = 0.02243 \) per second.
2. After 90 seconds the product \( kt = 0.02243 \times 80 = 1.795 \).

3. The final activity is therefore \( 750 \times e^{kt} = 750 \times e^{1.795} = 750 \times 0.166 = 125 \text{ Bq} \).

Example 4: The activity of a sample of \(^{239}\text{Pu}\) is 30 \( \mu \text{Ci} \) (30 micro curies). How long need we wait for the activity to drop to 1 \( \mu \text{Ci} \)?

1. The half life of \(^{239}\text{Pu}\) is 24,110 years. From this figure we can calculate the decay constant, \( k = \ln(2)/T_{1/2} = \ln(2) / 24110 = 2.875 \times 10^{-5} \) per year. (Note: there is no need to convert 24,110 years to seconds, as long as we remember that the time scale for the answer will be measured in years.)

2. \( N_o/N = A_o/A = 30 \mu \text{Ci} / 1 \mu \text{Ci} = 30 \)

3. The time we need to wait is then \( (1/k) \times \ln (N_o/N) = (1/2.875 \times 10^{-5}) \times \ln(30) = 118,300 \) years.

Carbon dating

The stable isotope of carbon is \(^{12}\text{C}\), with six protons and six neutrons. Nearly all of the carbon around us (including that in our own bodies) is \(^{12}\text{C}\). However, there is a small fraction of \(^{14}\text{C}\) in the environment. Since it is radioactive it is constantly decaying to \(^{14}\text{N}\) by beta emission

\[ ^{14}_6\text{C} \rightarrow ^{14}_7\text{N} + ^0_\text{e} + ^0_\nu + \text{energy} \]

However, it is also being constantly produced by neutrons bombarding the atmosphere as one component in a flux of particles known as cosmic rays. The process that creates \(^{14}\text{C}\) is the nuclear reaction

\[ ^0_\text{n} + ^{14}_7\text{N} \rightarrow ^{14}_6\text{C} + ^1_\text{p} \]

The fraction of \(^{14}\text{C}\) in the environment is determined as a balance between their rate of production by neutrons in the atmosphere and their loss rate through beta decay. Since the cosmic ray flux is constant the fraction of \(^{14}\text{C}\) in the environment is constant at about one \(^{14}\text{C}\) atom for every trillion \((10^{12})\) \(^{14}\text{C}\) atoms, at least over the last 100,000 years. (See below for an exception to this rule).

Example: When fresh a sample of organic material would be expected to have an activity of 5.60 Bq. If the measured activity is 0.34 Bq, how old is the sample?
1. The half life of $^{14}$C is 5,730 years. From this figure we can calculate the decay constant, $k = \ln(2)/T_{1/2} = \ln(2)/5730 = 1.21 \times 10^{-4}$ per year.

2. The time that has elapsed is then $(1/k) \ln (N_o/N) = (1/1.21 \times 10^{-4}) \ln(5.6/0.34) = 23,000$ years.

**Limitations of $^{14}$C dating**

As important and reliable as carbon dating is, it does have its limitations. First of all it can only be used to date organic material which once was alive, wood for example. It cannot be used to date non-organic material such as rock. Secondly, it is can only be used to date material from living organisms which died at least 200 years ago but not more than about 50,000 years ago.

The lower limit is set by the need for the activity of the sample to have changed appreciably from its value at the time that the organism died. With a half life of 5730 years the activity of $^{14}$C will change by about 2.5% after 200 years. A change of only 2.5% is difficult to detect, let alone measure accurately.

The upper limit is set by the need to have some activity left to measure. After 50,000 years the activity of a sample will be only about 0.2% of its original value. Since many samples are small, and so would have yielded only a small activity when fresh, after 50,000 years the activity has become so low that it is now difficult to even detect it.

The long term variation of the $^{14}$C concentration in the atmosphere is a second limitation to the use of carbon dating for very old samples. If the $^{14}$C ratio in the atmosphere were truly constant then calculation of the age of a sample by the method outlined above would be easy. However, it known that the atmospheric concentration of $^{14}$C varies slightly. The fluctuations are not large, of the order of 10%, but that is enough to affect the calculated age of a 5,000 year old sample by a few hundred years. Comparison of the results of carbon dating of samples whose age is known from other methods (dendrochronology and lake bed sediments) allow us to “calibrate” the carbon dating method, and therefore to make the appropriate correction when calculating the ages of samples whose ages are otherwise unknown. A literature search will yield many tests which explain the details of the calibration procedure.

**Recent $^{14}$C levels**

Although using the decay in the activity of $^{14}$C is not possible for very young samples, its radioactive properties allow us to detect very small amounts of $^{14}$C very easily. This can
be exploited in a second method of dating organic samples, but in this case of very recent origins.

As a result of nuclear weapons testing in the mid-1950’s there was an increase in the amount of $^{14}$C in the atmosphere. The $^{14}$C level peaked in the mid 1960’s, at about twice its equilibrium levels, and has steadily decreased since then as $^{14}$C is removed from the atmosphere, principally into the oceans. By the 21st century it had fallen almost back to its equilibrium value.

This does give us a second means of using $^{14}$C for dating organic material, but in this case not be using its decay curve. The few decades that have passed since nuclear testing (particularly aboveground testing) was banned corresponds to only a small fraction of the half life, and any sample which is so recent will have the same amount of $^{14}$C in it now as it had when that organism died. By comparing that amount with the known atmospheric concentration of $^{14}$C gives us a simple measurement of the age of the sample. In this case the radioactive properties of $^{14}$C make detection and measurement of this isotope easy. There are some complications to this method (for example it makes a difference whether the sample comes from the northern or southern hemisphere) which are described in the literature. Nevertheless this method of carbon dating has proved useful\(^1\).

**Other isotopic dating methods**

Some other useful dating methods based on radioactive nuclei are given in the table below. Note that the two uranium to lead methods are each a series of radioactive decays starting with a naturally occurring uranium isotope and ending with lead (see radioactive series above). Since the half lives of the intermediate isotopes in each series are much shorter than those of the uranium isotopes the half life associated with the whole series is that of the uranium isotope. Also, these dating methods are usually performed together and so give a common date.

<table>
<thead>
<tr>
<th>Method</th>
<th>Process</th>
<th>Half life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium lead dating</td>
<td>$^{238}$U to $^{206}$Pb</td>
<td>4.47 billion years</td>
</tr>
<tr>
<td>Uranium lead dating</td>
<td>$^{235}$U to $^{207}$Pb</td>
<td>700 million years</td>
</tr>
</tbody>
</table>

### Conversion of mass to energy

In any radioactive decay process the mass of the parent nucleus is always more than the combined mass of all the products. The lost mass is converted to energy, principally the kinetic energy of the lightest product. If for any proposed radioactive process the mass does not decrease, then that reaction cannot occur.

Once the mass loss has been computed then the energy can be found using the conversion 1 amu = 931.4 MeV.

The masses below have not been corrected for the masses of the electrons. In the case of alpha decay the corrections to the parent and daughter particles cancel. However in the case of beta decay (either electron or positron emission) the answers below should strictly speaking be corrected by the mass of an electron (2 electrons for positron emission). However, these corrections are small, and will be disregarded in a class at this level. Our numbers might be slightly 'off', but the same concepts still apply.

**Example 1: Alpha Decay of \(^{187}\text{Bi}\)**

The \(^{187}\text{Bi}\) isotope of bismuth is an alpha emitter, producing Thallium as the daughter

\[ ^{187}\text{Bi} \rightarrow ^{183}\text{Tl} + ^{4}\text{He} \]

The isotopic masses of the two nuclei and the alpha particle are

- \(^{187}\text{Bi}\) - 186.993 amu
- \(^{183}\text{Tl}\) - 182.983 amu
- \(^{4}\text{He}\) - 4.0026 amu
The total mass of the daughter and alpha particle is therefore 182.983 + 4.0026 = 186.9856 amu, which is less than the mass of the original \(^{187}\)Bi parent nucleus (186.993 amu). There is a mass loss of 186.993 - 186.9856 = 0.0074 amu. This lost mass appears as energy, principally kinetic energy of the alpha particle. When converted this corresponds to an energy of 6.9 MeV.

Note that the mass loss is a very small fraction of the total mass, which is why accurate nuclei masses are needed in these calculations.

**Example 2:** Beta decay of \(^{27}\)Mg

\(^{27}\)Mg is a beta emitter, producing \(^{27}\)Al

\[ ^{27}\text{Mg} \rightarrow ^{27}\text{Al} + e^- \]

The masses for this reaction are

- \(^{27}\text{Mg} \) - 26.9843 amu
- \(^{27}\text{Al} \) - 26.9815 amu
- \(e^-\) - 0.0005 amu

The total mass of the daughter and beta particle is therefore 26.9815 + 0.0005 = 26.9820 amu, again less than the mass of the original \(^{27}\)Mg parent. There is a mass loss of 26.9843 - 26.9820 = 0.0023 amu. This lost mass appears as energy, principally kinetic energy of the beta particle. When converted this corresponds to an energy of 2.14 MeV.

**Example 3:** Beta (positron) decay of \(^{18}\)F

\(^{18}\)F is also a beta emitter, but in this case it emits a positron, as opposed to an electron. (A positron is exactly the same as a n electron, except that it has a positive electrical charge, rather than a negative one.)

\[ ^{18}\text{F} \rightarrow ^{18}\text{O} + e^+ \]

The masses for this reaction are

- \(^{18}\text{F} \) - 18.0009 amu
- \(^{18}\text{O} \) - 17.9992 amu
- \(e^+\) - 0.0005 amu (same as for the electron)
The total mass of the daughter and beta particle is therefore 17.9992 + 0.0005 = 17.9997 amu, again less than the mass of the original $^{18}\text{F}$ parent. There is a mass loss of 18.0009 - 17.9997 = 0.0012 amu. This lost mass appears as energy, principally kinetic energy of the positron. When converted this corresponds to an energy of 1.12 MeV.

Example 4: Alpha Decay of $^{53}\text{V}$ - a forbidden reaction

The $^{53}\text{V}$ isotope of vanadium is actually a beta emitter. But, for the moment let us suppose that it might be an alpha emitter, producing scandium as the daughter

$$^{53}\text{V} \rightarrow ^{49}\text{Sc} + ^{4}\text{He}$$

The isotopic masses would be

- $^{53}\text{V}$ - 52.9443 amu
- $^{49}\text{Sc}$ - 48.95 amu
- $^{4}\text{He}$ - 4.0026 amu

The total mass of the daughter and alpha particle is therefore 48.95 + 4.0026 = 52.9526 amu, which in this case is more than the mass of the original $^{53}\text{V}$ parent. There would have to be a mass gain of 52.9526 - 52.9443 = 0.0083 amu. Since there is no way for this to happen, the alpha decay of $^{53}\text{V}$ is forbidden.