Nuclear Reactions

Introduction

As we saw in the notes "Radioactive sources as power stations" the use of radiative sources is limited to just those applications which require very little power, and for which the availability of electricity is limited. When it comes to large scale electricity generation, for example to supply a city, the spontaneous process of radioactive decay is much too slow, we have to find a way to speed it up. Since there is no known way of affecting the decay constant (or equivalently the half life) then we shall introduce a particle to make the nucleus release energy quickly, a nuclear hammer if you like.

Introducing a new particle means that we will now have two particles before the reaction occurs, and at least two (and different) particles after it. Such processes are termed nuclear reactions. We have already seen on before, when discussing carbon dating. The formation of ^{14}C in the atmosphere is a nuclear reaction involving atmospheric nitrogen and protons contained in cosmic rays

$$^{1}_{0}n + \ ^{14}_{7}N \rightarrow \ ^{1}_{1}p + \ ^{14}_{6}C$$

In this example there is a simple interchange of a proton for a neutron.

Options

We are looking for a reaction which is capable of releasing a large amount of energy (o the nuclear scale), which can be sustained at a rapid rate, and in the case of a power station can be controlled. In the case of a nuclear weapon the last of these conditions is not relevant. That leaves us with two possibilities

- Colliding two particles together and splitting at least one into smaller pieces. This process is termed *fission*. It will produce two (sometimes more) elements of smaller atomic number.
- Colliding two particles together and making them 'stick' together in a process known as *fusion*. It will produce an element of larger atomic number.

In both cases the source of energy is the binding energy needed to hold the nucleus together, which in turn comes from the loss of mass when the mass of the atom is compared to the sum of the masses of all its individual protons, neutrons, and electrons (see notes on “Structure of the atom”). For either of these processes to occur we must be able to extract energy, which means that the mass must decrease even further as a result of the reaction. Referring again to the notes on the “Structure of the atom” the graph of the binding energy (and also mass loss) as a function of atomic number peaks at Z=26, the element iron. It therefore follows that

- fission reactions are only possible for elements beyond iron in the periodic table (Z>26). In practice only the isotopes ^{235}U and ^{239}Pu are used. Other isotopes with Z>26 can undergo fission reactions, but the probability of inducing a reaction is too low to be practical.

- Fusion reactions are only possible for elements up to iron in the periodic table (Z<26). Hydrogen is easily the most preferred, although the isotopes deuterium (^{2}H) and tritium (^{3}H) are better than normal hydrogen (^{1}H). Other elements are not practical in Earthly reactors, but they are important for energy generation in the interiors of stars.

Similarities with radioactivity

Although nuclear reactions are fundamentally different to radioactivity, and have some important differences, there are also some features which they have in common

- The total atomic number has to be conserved. In other words the total charge before the reaction must equal the total charge after the reaction.

- The total mass number has to be conserved. In other words the total number of nucleons (protons and neutrons) before the reaction must equal the total number of nucleons after the reaction.

- An exothermic (energy producing) reaction occurs because the mass decreases, and the energy that is produced is given by Einstein's famous equation $E = mc^2$. In units of MeV (energy) and amu (mass) this again reduces to $E = m * 931.4$[(1)].

Reaction cross sections

In discussing nuclear reactions an important concept is the probability that a reaction will occur given the right conditions. This probability is usually expressed in terms of a cross

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1 Since there is now two incoming particles, with some sort of relative speed, if that speed is high enough there is the possibility of endothermic (energy absorbing) reactions in which extra mass is produced. However, the only significant environment in which this occurs is a supernova, the explosive death of a massive star.

section. It is defined such that the product of the cross section, the relative speed of the reagents, and their densities gives the rate at which the reaction will occur. The details of this definition are not important to us. Just remember that a high cross section means that the reaction is likely, a low cross section means that the reaction almost never occurs.

Fission

In the fission reaction a large nucleus (such as ^{235}U) is hit with a small projectile in order to induce the large nucleus to split into pieces, usually two large fragments plus a number of small ones. The choice of projectile is really determined by the electrostatic forces between the atom and the projectile. Although the electrons of the atom play no role in nuclear reactions the incoming projectile has to make its way through them if it is to reach the nucleus. That means that electrons would be a very poor choice. The electrostatic repulsion between electrons would preclude almost all electron projectiles from reaching the nucleus. Protons are attracted to the outer electrons, and would make their way through them without problem, but the electrostatic forces would repel them from the nucleus. That leaves neutrons, which are unaffected by the electrostatic force, and so are able to penetrate through the outer electrons all the way to the nucleus. The fission of ^{235}U is therefore the (schematic) reaction

$$^{1}_{0}n + ^{235}_{92}U \rightarrow \text{fission fragments}$$

The identity of the fission fragments is statistical, no hard and fast rule exists to predict the products akin to the rules for predicting the daughters in radioactive decay. The only restriction is that the atomic and mass numbers have to be conserved. Since small fragments are also produced in the fission reaction, satisfying this rule is possible with many different possible product isotopes. For example, one possible fission reaction is

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + ^{1}_{0}n + ^{1}_{0}n$$

This does preserve the atomic number (total charge = 92) and mass number (total number of nucleons = 236) between the reactants and the products. Remember that this is only one example out of many, you can write down dozens of other reactions, with different product isotopes which also have a total charge of 92 and a total mass number of 236.

Energy production in fission reactions

Fission reactions such as the one above can be used as sources of energy only because the mass decreases as a result of the reaction. The calculation is identical to the calculation that we performed to find the energy released as a result of either α or β decay, except that we now have two particles before the reaction occurs. For the reaction

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + ^{1}_{0}n + ^{1}_{0}n$$

Particle	Mass (amu)
Neutron (^{1}n)	1.008665
^{235}U	235.0439
^{140}Xe	139.9216
^{94}Sr	93.9154

the masses are given in the table to the right. The total mass of the reacting particles ($^{1}_{0}n$ + $^{235}_{92}U$) is 236.0526 amu. After the reaction the product particles have a total mass of 235.8543 amu. The is therefore a mass loss of 0.1983 amu, and an energy gain of 0.1983 * 931.4 = 185 MeV[(2)].

Remember that this reaction is only one possible reaction out of many. There are dozens of possible product combinations, and for each one the energy released is calculated in the same way as for the one above. Typical values are in the same range, around 200 MeV.

Try one yourself

Another possible product combination gives the fission reaction

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + ^{1}_{0}n + ^{1}_{0}n + ^{1}_{0}n$$

In this reaction, how much energy is released? (Answer 173 MeV)

Fission and ^{239}U

Since the impact of a neutron on a ^{235}U nucleus can lead to fission, and since natural uranium also contains ^{238}U (natural ore is 99.3% ^{238}U and only 0.7% ^{235}U) an obvious question what happens when a neutron impacts a ^{238}U nucleus.

To answer this question we shall first look at a detail of the above reaction. It shows the initial and final particles, but there is a temporary intermediate step. The first event is the absorption of the neutron by the uranium nucleus, and then fission of the unstable ^{236}U nucleus a fraction of a second later

2 The total yield is usually a little higher. The fission process produces a variety of different isotopes, many of which are radioactive, and usually with relatively short half lives. They will at some later time decay, adding yet more energy to the equation.

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + ^{1}_{0}n + ^{1}_{0}n$$

The intermediate ^{236}U nucleus is not important to understanding how this reaction occurs, and how it produces energy. However, in the case of ^{238}U the reverse is true. When it absorbs a neutron it forms a ^{239}U nucleus,

$$^{1}n + ^{238}U \rightarrow ^{239}U$$

which rarely undergoes fission(3). However, the ^{239}U nucleus is radioactive (a β^- emitter) to form ^{239}Np, which is also a β^- emitter forming ^{239}Pu as the daughter

$$^{1}n + ^{238}U \rightarrow ^{239}U \rightarrow ^{239}Np + ^{o}e^-$$

$$^{239}Np \rightarrow ^{239}Pu + ^{o}e^-$$

Even though the uranium and neptunium isotopes are not useful in a fission device, the plutonium isotope ^{239}Pu is. It is a synthetic isotope, plutonium is not found in nature(4). The above pair of equations represent to process by which we make it.

Chain Reactions

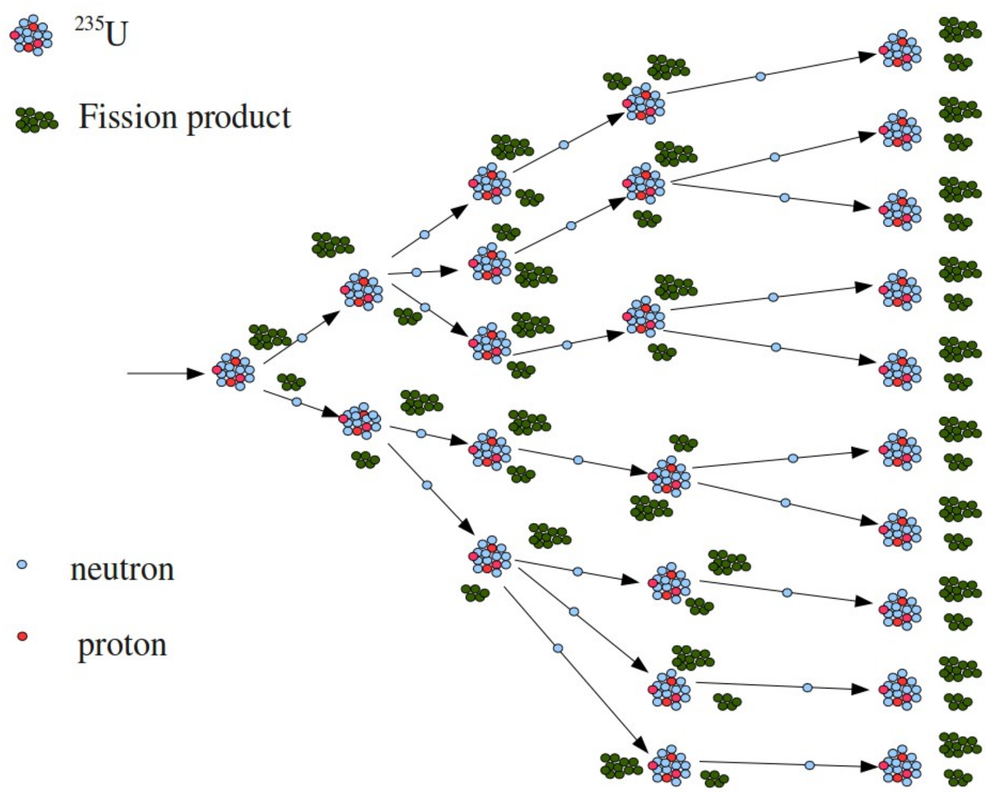
The fission reaction is capable of supplying large quantities of energy, enough to make a power station, providing that there is a sufficient supply of neutrons. We can easily estimate the number needed if we know the design capability of the power station. Suppose, for example, that we wish to build a 500 MW power station. If we use a figure of 200 MeV per fission reaction, then

- Energy per fission reaction = 200 MeV * 1.6 x 10^{-13} J/MeV = 2.6 x 10^{-11} J
- Required power level = 500 MW = 5 x 10^{8} W = 5 x 10^{8} J/s
- Number of fission reactions needed = (5 x 10^{8} J/s) / (2.6 x 10^{-11} J per reaction) = 2 x 10^{19} reactions per second.

3 The cross section (probability) of fission occurring for ^{238}U using slow neutrons is a million times smaller than it is for ^{235}U. As the speed of the neutron increases the cross section for ^{235}U falls, and that for ^{238}U rises, making the ratio is a bit more favorable, but even for fast neutrons the cross section for ^{238}U is still only about ⅓rd that for ^{235}U. (*Physics of the Atom*, by Wehr, Adair, and Richards, Addison Wesley)

4 All its isotopes have half lives which are much shorter than the age of the Earth.

Each fission reaction requires one neutron, so we also need about 2 x 10^{19} neutrons per second. This is a large number, much more than is readily available. We need to make our own inside the reactor, and the source of these 'new' neutrons is going to be the neutrons formed by the fission process itself (see the reactions above). On average, when ^{235}U undergoes fission 2.4 neutrons are produced. When ^{239}Pu undergoes fission almost 3 neutrons are produced on average.



When the neutrons produced at one time are used to cause more fission at a later time we have a *chain reaction*. It is illustrated in the diagram above. One stray neutron starts the process going by causing fission in one ^{235}U nucleus. (The process is the same if we ^{239}Pu is used.) After that the reaction proceeds by itself, with the need for more external neutrons, all the neutrons that are needed are produced within the sample of uranium. As can be seen in the diagram the number of neutrons increases rapidly.

If we assume the figure for uranium, that is there are 2.5 neutrons produced per fission reaction, and that we don't lose any, then after two stages of the chain reaction we will have 2.5^2= 6.25 neutrons, after three stages we will have 2.5^3 = 15.6 neutrons, and so on. In general after N stages we will have 2.5^N neutrons. From that it is easy to figure out how many stages it will take to reach the required number of neutrons (2×10^{19}). You might be surprised how few, but the exponential rise of the function 2.5^N means that the number does rise very fast. It takes only 49 chain reaction stages to reach the required number ($2.5^{49} = 3.2 \times 10^{19}$). Since each stage of the chain reaction takes about 10^{-14} s (the time it take for the intermediate ^{236}U nucleus to split) 48 stages takes only about a trillionth of a second. We should point out at this stage that the is a large loss of neutrons also (see notes on power stations) which we have ignored. This will extend the time required to build up the number of neutrons to the required level, but even then it is still a small fraction of a second.