Solving Assignment 3

A. Reviewing Strategies

The preamble to the assignment laid out the basic strategy for answering the questions in this assignment, that is to determine the number of "items" which come in a "package".

$$\text{number of items} = \frac{\text{number of items}}{\text{package}} * \text{number of packages}$$

You can define the item and the package as being anything you want. That must be the first step in your answer to each problem. Miss this step and you're "flying blind." Follow this procedure each and every time:

- Establish (and write down) the logic for the problem by identifying the "item" and the "package".
- Insert the known numbers in the equation.
- Last of all, do the arithmetic.

B. Solutions to Assignment 3

1. One ***chemical*** reaction typically releases about 2 eV of energy. Convert that to Joules. (You will need this later.)
   a. This problem is a conversion of units, identical to two of the examples from the assignment
      i. "item" is a Joule
      ii. "package" is an eV

$$\text{number of Joules} = \frac{\text{number of Joules}}{\text{eV}} * \text{number of eV}$$

   b. Substitute the known numbers

$$\text{number of Joules} = \frac{1.6 \text{ x } 10^{-19}\text{ J}}{\text{eV}} * 2\text{ eV}$$

   c. Finally do the arithmetic, that is multiply the numbers together to get the final answer that 2eV is equal to $3.2 \text{ x } 10^{-19}$ J. (Remember this number, you need it for question 3.)

2. One ***nuclear*** (fission) reaction typically releases about 5 MeV of energy. Convert that to Joules.
   a. This problem is also conversion of units, but with a much higher amount of energy
      i. "item" is still a Joule
      ii. "package" is now an MeV (the same as a million eV.)

$$\text{number of Joules} = \frac{\text{number of Joules}}{\text{MeV}} * \text{number of MeV}$$

   b. Substitute the known numbers

$$\text{number of Joules} = \frac{1.6 \text{ x } 10^{-13}\text{ J}}{\text{MeV}} * 5\text{ MeV}$$

c. Finally do the arithmetic, again by multiplying the numbers together to get the final answer that 5 MeV is equal to 8 x 10^{-13} J. (Remember this number, you need it for question 4.)

3. If you want to build a 100 MW (10^8 J/s) power station based on molecular ***chemical*** processes
   a. How many reactions per second are needed?

      i. For this problem the energy comes from individual chemical reactions, one molecule at a time.
         - “item” is the energy (measured in Joules)
         - “package” is a chemical reaction

$$\text{Energy} = \frac{\text{Energy}}{\text{reaction}} * \text{number of reactions}$$

      ii. Substitute the known numbers, using the energy from question 1 (to keep the energy units consistent). For each second of operation we need 10^8 J of energy, and we get 3.2 x 10^{-18} J from each reaction. The final term (the number of reactions) is the unknown quantity

$$10^8\,\text{J} = \frac{3.2 \text{ x } 10^{-19}\,\text{J}}{\text{reaction}} * \text{number of reactions}$$

      iii. Finally do the arithmetic, this time by rearranging the equation (divide each side by 3.2 x 10^{-19}). The final result is that you need 3.13 x 10^{26} reactions (every second).

   b. If each molecule has a mass of 2 x 10^{-26} kg, how much fuel is needed in a day?
      i. “item” = mass (in kg)
      ii. “package” = molecule

$$\text{mass} = \frac{\text{mass}}{\text{molecule}} * \text{number of molecules}$$

      iii. The number of molecules is equal to the number of reactions (you can only use a given molecule once, so only one reaction per molecule)

$$\text{mass} = \frac{2 \text{ x } 10^{-26}\,\text{kg}}{\text{molecule}} * 3.13 \text{ x } 10^{26}\,\text{molecules every second}$$

      iv. Finally the arithmetic gives is the value of 6.26 kg every second.
      v. The question asks for the mass per day, so we need the number of seconds in a day. That is 86,400 (60*60*24), and so.

$$\frac{\text{mass}}{\text{day}} = \frac{\text{mass}}{\text{second}}\,\frac{\text{seconds}}{\text{day}} = 86{,}400 * 6.26 = 540{,}000\,\text{kg per day}$$

      (1000 kg is about 1 ton, so that is 540 tons per day.)

4. If you want to build a 100 MW power station based on ***nuclear*** processes
   a. How many reactions per second are needed?

i. This problem is identical to previous once, except that the energy comes from individual *nuclear* reactions, one *nucleus* at a time.
   - "item" is the energy (measured in Joules)
   - "package" is a *nuclear* reaction

$$\text{Energy} = \frac{\text{Energy}}{\text{reaction}} * \text{number of reactions}$$

ii. Substitute the known numbers, using the energy from question 2 (again to keep the energy units consistent). For each second of operation we need 10^8 J of energy, and we get 8 x 10^{-13} J from each reaction. The final term (the number of reactions) is the unknown quantity

$$10^8\,\text{J} = \frac{8 \text{ x } 10^{-13}\,\text{J}}{\text{reaction}} * \text{number of reactions}$$

iii. Finally do the arithmetic (divide each side by 8 x 10^{-13}). The final result is that you need 1.25 x 10^{20} reactions (every second).

b. If each nuclear mass is 238*1.66x10^{-27} kg, how much fuel is needed in a day?
   i. "item" = mass (in kg)
   ii. "package" = nucleus

$$\text{mass} = \frac{\text{mass}}{\text{nucleus}} * \text{number of nuclei}$$

   iii. The number of nuclei is equal to the number of reactions (you can only use a given nucleus once, so only one reaction per molecule)

$$\text{mass} = \frac{238 * 1.66 \text{ x } 10^{-27}\,\text{kg}}{\text{nucleus}} * 1.25 \text{ x } 10^{20}\ \text{nuclei every second}$$

   iv. Finally the arithmetic gives is the value of 4.9 x 10^{-5} kg every second.
   v. As before multiply by the number of seconds in a day to get the mass needed in a day

$$\frac{\text{mass}}{\text{day}} = \frac{\text{mass}}{\text{second}}\frac{\text{seconds}}{\text{day}} = 4.9 \text{ x } 10^{-5} * 86{,}400 = 4.3\ \text{kg per day}$$

(1 kg is about 2.2 pounds, so that is only about 10 pounds per day. Compare that with the result we got for chemical powered reactor, over 500 tons per day.)

5. Which releases more energy, a 20 kiloton nuclear bomb (about the size of each bomb dropped on Japan) or a 250 MW nuclear power station operating for 30 years? (In each case find the energy in Joules, so that you can directly compare them. A kiloton is a unit commonly used to gauge the energy released by weapons, equivalent to 4.184 x 10^{12} Joules.)
   a. For the bomb, this is a unit conversion problem, logically identical to questions 1 and 3
      - "item" is a Joule
      - "package" is a kiloton

$$\text{number of Joules} = \frac{\text{number of Joules}}{\text{kiloton}} * \text{number of kilotons}$$

ii. Substitute the known numbers

$$\text{number of Joules} = \frac{4.184 \text{ x } 10^{12} \text{ J}}{\text{kiloton}} * 20 \text{ kilotons}$$

iii. Finally do the arithmetic, that is multiply the numbers together to get the final answer that 20 kilotons is equal to 8.4 x 10^{13} J.

b. For the power station the calculation is logically the same as the last parts of questions 3 and 4. We will need the number of seconds in 30 years. That is equal to 60*60*24*365*30 = 9.5 x 10^{8}.

$$\text{energy in 30 years} = \frac{\text{energy}}{\text{second}} * \text{number of seconds in 30 years}$$

$$\text{energy in 30 years} = \frac{250 \text{ x } 10^{6} \text{ Joules}}{\text{second}} * 9.5 \text{ x } 10^{8} \text{ seconds}$$

and so the energy generated is 2.4 x 10^{17} Joules.

Comparing the two results, the energy release by the power station is much higher then that released by the bomb.