Schroedinger's Equation

Introduction

Hamiltonian

Schroedinger's formulation of quantum mechanics starts with the classical mechanics concept of the Hamiltonian. Conceptually we can think of the Hamiltonian as being the sum of the kinetic energy and the potential energy. If these two terms are added algebraically we get the total energy. Schroedinger replaces the algebraic mathematics with operator mathematics

$$\mathscr{H} = \check{T} + \check{U}$$

Schroedinger also gives us a means of expressing the kinetic energy ($\check{T}$) and Hamiltonian ($\mathscr{H}$) operators

- $\check{T} \rightarrow - \hbar^2/(2m)\ \nabla^2$
- $\mathscr{H} \rightarrow i\hbar\ \partial\ /\partial t$

On the other hand the potential operator has to be determined separately for each problem to which his equation is applied.

Wave functions

In Schroedinger's quantum mechanics a complete description of the system is contained within a wave function $\Psi(\mathbf{r},t)$ from which all of the properties can be found. The wave function is the solution to the equation

$$\mathscr{H}\ \Psi = i\hbar\ \partial\Psi\ /\ \partial t$$

which is Schroedinger's Equation in its most general form.

Energies

The form of Schroedinger's equation given above is known as the time dependent form. For those cases where the potentail energy ($\check{U}$) is not time dependent the wave function can be separated into a factor which only depends on poisition multiplied by ine which only depends on time

$$\Psi(\mathbf{r},t) = \psi(\mathbf{r})\ e^{iEt/\hbar}$$

where ψ is the solution of the equation

$$\mathscr{H}\ \psi = E\ \ \psi$$

which is the time independent form of Schroedinger's Equation.

The parameter E which appears on the right hand side of the time independent Schroedinger Equation is the total energy of the system, the equivalent of algebraically adding the different energies of a classical system.