Quantum Mechanics Concepts

Introduction

For any physical property which can be described quantum mechanically by the operator $\hat{O}$, if there exists one (or more) wave function ψ for which

$$\hat{O}\ \psi = K\ \psi$$

then any measurement will always yield the value K. Otherwise, a measurement could yield any one of a number of different values. The best we can do is predict the average of all possible measurements, known as the expectation value.

Examples

Property	Classical	QM	
Momentum	$p = m\mathbf{v}$	$p \rightarrow -i\hbar\nabla$	$p\ e^{ikx} = \hbar k\ e^{ikx}$
Energy Hamiltonian	$\mathscr{H} = p^2/2m + V$	$\mathscr{H} \rightarrow -\hbar^2\nabla^2/2m + V$	$\mathscr{H} \sin(n\pi x/a) = h^2n^2/8ma^2 \sin(n\pi x/a)$
Angular Momentum	$L = r \times p$	See below	$L^2\ Y_\ell^{\lvert m\rvert}(\theta,\varphi) = \ell(\ell+1)\hbar^2 Y_\ell^{\lvert m\rvert}(\theta,\varphi)$
(Angular Momentum)$_z$	$L_z = xp_y - yp_x$	$L_z \rightarrow i\hbar\ \partial/\partial\varphi$	$L_z Y_\ell^{\lvert m\rvert}(\theta,\varphi) = m\hbar\ Y_\ell^{\lvert m\rvert}(\theta,\varphi)$
(Angular Momentum)$_x$	$L_x = yp_z - zp_y$		$L_z Y_\ell^{\lvert m\rvert}(\theta,\varphi) \neq K\ Y_\ell^{\lvert m\rvert}(\theta,\varphi)$
Spin Angular Momentum	No classical equivalent	S	$S^2\ \psi(n,\ell,m_\ell,s,m_s) =$ $s(s+1)\hbar^2\ \psi(n,\ell,m_\ell,s,m_s) = 3\hbar^2/4$
(Spin Angular Momentum)$_z$		S_z	$S_z\ \psi(n,\ell,m_\ell,s,m_s) = m_s\hbar\ \psi(n,\ell,m_\ell,s,m_s)$

$$L^2 \rightarrow -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$