Multibeam Interference and Diffraction Gratings

Introduction

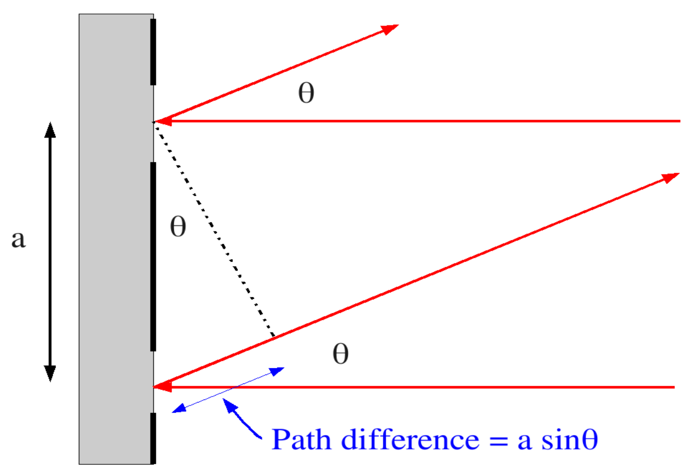
$$I = I_o \frac{\sin^2\left(\frac{1}{2} N \delta\right)}{\sin^2\left(\frac{1}{2}\delta\right)}$$

The starting point is the general expression for multibeam interference

The next step is to replace the generic term δ, the relative phase difference between two adjacent beams.

Relative phase difference for the diffraction grating

For the diffraction grating the relative phase difference in caused by the different paths that the two beams have to traverse before arriving at the point where the interference is observed. We shall start by assuming that the light falls on to the grating at normal incidence. In that case the phase difference is caused only by the outgoing beams.



The diagram shows two beams diffracting at an angle θ from neighboring lines of a diffraction grating. The triangle in the diagram shows that the extra path length traversed by the lower beam is equal to asin(θ) where a is the spacing between lines of the grating. This corresponds to a fraction asin(θ) / λ of a wavelength, and any one wavelength is a phase change of 2π. The phase change caused by the path difference is therefore

$$\delta = \frac{2\pi a \sin(\theta)}{\lambda}$$

Maxima

From the treatment of multibeam interference, a maximum in the intensity of the interference pattern is observed whenever δ = 2pπ. Substituting into the equation above gives the angular positions of the maxima

$$\sin(\theta) = \frac{p\lambda}{a}$$

In this equation p is known as the ***order number*** (or simply just order).

Typical numbers

Gratings are specified by the number of lines per millimeter (l/mm or g/mm). Typical values are 600 l/mm, 1200 l/mm, and 1800 l/mm. The line spacing is the inverse of this number. For example, if a grating has 1200 l/mm, then the line spacing is

$$a = \frac{1}{1200 l/mm} = 8.33 \times 10^{-4}\,mm = 8.33 \times 10^{-7}\,m$$

Note this is comparable to the wavelength of light, One consequence of this is the maximum order number that can be observed is usually limited. For example, suppose the wavelength of the incident light is 4×10^{-7}m = 4000 Å. Then for each value of the order number the angular position of the maxima are

p	θ
1	$\sin^{-1}(0.48) = 28.7°$
2	$\sin^{-1}(0.96) = 73.7°$
3	$\sin^{-1}(1.44)$ which has no solution

There are only two orders observed.

Overlapping orders

Overlapping orders occurs when two lines of different wavelength occur at the same position of the interference spectrum but with different values of the order number. The presence of overlapping orders can complicate the analysis of a spectrum unless there is some way to determine what is the order number of each of the spectral peaks (maxima) that are seen.

Whereas overlapping orders can occur for a diffraction grating it is not a serious problem(1). Since only small values of the order number are seen the corresponding wavelengths have to be very different. For example, supposing that in first order there is a maximum corresponding to a wavelength towards the end of the visible spectrum, that is at about 7000 Å. An adjacent line might also being seen in first order, with a wavelength also close to 7000 Å, or it might be a line in second order but with a wavelength of 3500 Å. With such different wavelengths it is easy to discriminate between them. A wavelength of 3500 Å corresponds to the ultraviolet part of the spectrum. Insertion of a filter which blocks the ultraviolet but lets through red will immediately tell which lines are being seen in first order and which are being seen in second order.

Resolution

The resolution of any instrument is a measure of the minimum difference in wavelength of two lines which can still be resolved as separate, relative to their actual wavelength λ/dλ. A common criterion which is applied to quantify the resolution is that two lines can be seen as separate if the maximum for one wavelength is seen at the position of the first minimum of the other.

1 By way of contrast it is a serious problem for an etalon, perhaps the major disadvantage of these devices.

Let the line of wavelength λ be observed in the p^{th} order using a grating with N illuminated lines (red line in see figure 1.) Then the angular position of the first minimum after the maximum is given by

$$\delta = \frac{2\pi a \sin(\theta)}{\lambda} = 2\left(p + \frac{1}{N}\right)\pi$$

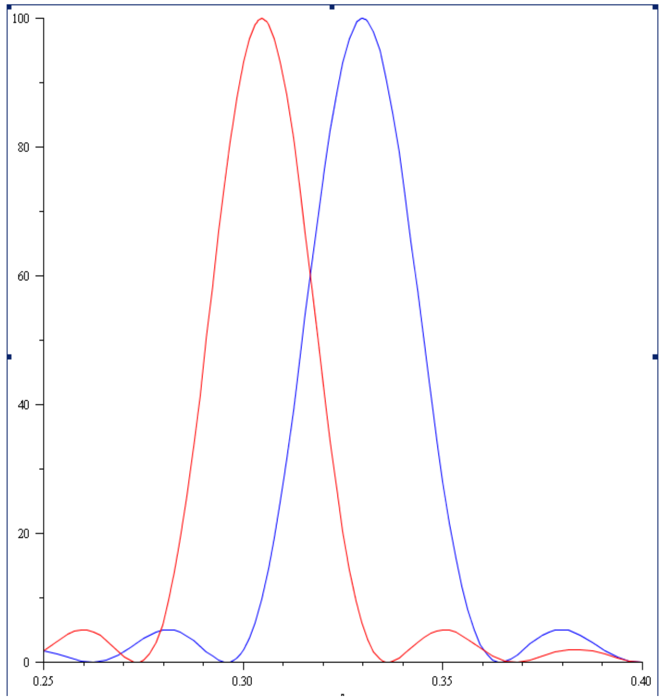


Figure 1

At the same position the line of wavelength λ' (blue line in figure 1) has its maximum, also in the p^{th} order[(2)].

$$\delta' = \frac{2\pi a \sin(\theta)}{\lambda'} = 2p\pi$$

It therefore follows that

$$\left(p + \frac{1}{N}\right)\lambda = p\lambda'$$

which can be rearranged to give

$$\frac{\lambda}{d\lambda} = \frac{\lambda}{\lambda - \lambda'} = pN$$

This shows that the maximum resolution of a diffraction grating depends only on the order number and the number of illuminated lines.

2 We shall ignore the possibility of overlapping orders, since for overlapping orders the difference in frequency would have to be large.