Data Analysis

Introduction

In this worksheet we will look at three different aspects of analyzing your data, and interpreting the results

- Propagation of errors
- Representing data in graphical form and using trends
- Using the data to estimate unknown values

Propagation of errors

1. The variables x, y, and z are related by the equation $z=4x^2+\frac{(x+y)}{\sqrt[3]{(x-y)}}$. Given that x = 0.1 ± 0.0025 and y = 0.03 ± 0.0008, what is z?

2. The grating equation is $n\lambda = d \sin\theta$. (Note: this assumes normal incidence which we will take to be true.) You measure the wavelength of light in second order (n=2) using is diffraction angle θ = 47° ± 1°. If d = (2 ± 0.01) x 10^{-6} m, what is the wavelength of the light? (Hint: be careful about assumptions when calculating derivatives.)

Representing data in graphical form and using trends

3. The table to the right contains measurements of x as a function of t. The known relationship between them is $x = A e^{\gamma t}$, where A and γ are to be determined.
   a. Rearrange the equation to be linear, and plot the graph.
   b. Find the unknown parameters A and γ, including their uncertainties. (In MS Excel use the data tab, select the Data Analysis option at the right end of the tool bar. From the list of options select Regression.

t	x
0	5.88
1	5.1
2	4.45
3	3.53
4	3.05
5	2.76
6	2.43
7	1.93
8	1.6
9	1.4
10	1.22
11	1.05
12	0.83
13	0.69
14	0.54
15	0.51
16	0.43

4. The table to the right contains measurements of x as a function of t.

   The known relationship between them is $x = e^{-\frac{(t - t_o)^2}{\delta^2}}$, where t_o and δ are to be determined.
   a. Rearrange the equation to be linear, and plot the graph.
   b. Find the unknown parameters t_o and δ, including their uncertainties. Note: there are two minor issues here to be dealt with. You will find out what they are once you plot the graph.

t	x
0	0.092
1	0.337
2	0.785
3	1.123
4	0.875
5	0.388
6	0.110
7	0.020
8	2.05×10^{-3}
9	1.09×10^{-4}
10	3.77×10^{-6}

Using the data to estimate unknown values

5. Using the results of question 3, find the value of x when t is equal to
   a. 7.7 ± 0.25
   b. 210 ± 2

6. The following is a list of lines observed in a spectrum, as a function of position on a printout of the data. For six of them the wavelength (λ, in nm) is known. Find the wavelength of the unknown line, including its uncertainty
   a. assuming that the spectrum is calibrated according to $\lambda = a + bx$
   b. assuming that the spectrum is calibrated according to $\lambda = a + bx + cx^2$. Excel will let you fit to a quadratic by writing $y = x^2$, and making a linear regression fit to the equation $\lambda = a + bx + cy$. Make a new column in your date sheet and fill it with the values of x^2. Then select the Data Analysis option at the right end of the Data tool bar, and from the list of options select Regression. In the dialog box that appears make the input data range span both the x and x^2 columns of your data.

x	λ
12.4	456
57.1	478
78.9	510
120.1	585
180.7	710
210	805
145 ± 0.3	?