These are all equations with constant coefficients. For each of them first find the complementary function, then the particular integral, and lastly apply the initial conditions.

1. $\frac{d^2x}{dt^2}-8\frac{dx}{dt}-20x=7e^{5t}$ given that x = $-\frac{1}{5}$ and $\frac{dx}{dt}=23$ each when t=0

$x=2e^{10t}-2e^{-2t}-\frac{1}{5}e^{5t}$

2. $\frac{d^2x}{dt^2}-4x=25\sin(t)$ given that x = 1 and $\frac{dx}{dt}=5$ each when t=0

$x=3e^{2t}-2e^{-2t}-5\sin(t)$

3. $\frac{d^2x}{dt^2}-3\frac{dx}{dt}-2x=e^{\frac{3t}{2}}$ given that x = 0 and $\frac{dx}{dt}=0$ each when t=0

$x=\frac{2}{17}e^{\frac{3+\sqrt{17}t}{2}}+\frac{2}{17}e^{-\frac{3+\sqrt{17}t}{2}}-\frac{4}{17}e^{\frac{3t}{2}}$

4. $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+5x=\sin(t)$ given that x = 0 and $\frac{dx}{dt}=1$ each when t=0

$x=\frac{9}{20}e^{t}\sin(2t)-\frac{1}{10}e^{t}\cos(2t)+\frac{1}{5}\sin(t)+\frac{1}{10}\cos(t)$