1. Find the polar representations of the following
   a. $z_1 = 1+i = \sqrt{2}\, e^{i(¼\pi+2n\pi)}$
   b. $z_2 = -1+i = \sqrt{2}\, e^{i(-¼\pi+2n\pi)}$
   c. $z_3 = 3+4i = 5\, e^{i(\alpha+2n\pi)}$ where $\alpha = \tan^{-1}(4/3) = 0.927$ rad
   d. $z_4 = -6-12i = \sqrt{180}\, e^{i(\alpha+2n\pi)}$ where $\alpha = \tan^{-1}(12/6) = 1.107 + \pi = 4.25$ rad
   e. $z_5 = -5+8i = \sqrt{89}\, e^{i(\alpha+2n\pi)}$ where $\alpha = \tan^{-1}(-8/5) = -1.01 + \pi = 2.13$ rad

2. Find the Cartesian forms of the following
   a. $5e^{i\pi/3} = 2.5 + 4.33\, i$
   b. $20e^{i4\pi/3} = -10 – 17.3\, i$
   c. $0.4e^{2.45i} = -0.309 + 0.255\, i$

3. If $z^6 = 64i$, find z.
   a. $z^6 = 64i = 2^6\, e^{i(½\pi+2n\pi)}$
   b. $z = 2\, e^{i(½\pi+2n\pi)/6}$
   c. If n = 0 then z = 1.93 + 0.517 i
   d. If n = 1 then z = 0.517 + 1.93 i
   e. If n = 2 then z = -1.414 + 1.414 i
   f. If n = 3 then z = -1.93 – 0.517 i
   g. If n = 4 then z = -0.517 -1.93 i
   h. If n = 5 then z = 1.412 – 1.414 i

4. If $z^3 + 6z^2 + 12z + 8 = (-1+i)/\sqrt{2}$, find z.
   a. Rearranging the equation becomes $(z+2)^3 = e^{i(¾\pi+\pi+2n\pi)}$
   b. $z+2 = e^{i(¾\pi+2n\pi)/3}$
   c. If n = 0 then z + 2 = 0.707 + 0.707 i, and so z = -1.293 + 0. 707 i
   d. If n = 1 then z + 2 = -0.965 + 0.259 i, and so z = -2.965 + 0. 259 i
   e. If n = 2 then z + 2 = 0.259 – 0.965 i, and so z = -1.741 – 0. 965 i

5. Solve the equation $z^2 - (2-3i)\, z + 10 + 24i = 0$ (This question is from last week. We can now find the solution using polar notation. Start with the partial answer from last week, and try to evaluate the square root part of the quadratic equation.)

   It can help to use the trig identities

$$\cos(\phi) = 2\cos^2\left(\frac{\phi}{2}\right) - 1 = 1 - 2\sin^2\left(\frac{\phi}{2}\right)$$

   From last week the solution is

$$z = \frac{(2-3i) \pm \sqrt{(-45-108i)}}{2} = \frac{(2-3i) \pm 3i\sqrt{(5+12i)}}{2}$$

   Task is to evaluation the term $\sqrt{(5+12i)}$.

   1. Write $5+12i = 13\, e^{i\varphi}$, where $\cos(\varphi) = 5/13$
   2. $\sqrt{(5+12i)} = \sqrt{13}\, e^{i\varphi/2} = \sqrt{13}\, \{\cos(\tfrac{1}{2}\varphi) + i \sin(\tfrac{1}{2}\varphi)\}$
   3. Using the trig identities $\cos(\varphi) = 2\cos^2(\tfrac{1}{2}\varphi) - 1 = 1 - 2\sin^2(\tfrac{1}{2}\varphi)$ we get
      a. $\cos(\tfrac{1}{2}\varphi) = 3/\sqrt{13}$
      b. $\sin(\tfrac{1}{2}\varphi) = 2/\sqrt{13}$
   4. $\sqrt{(5+12i)} = \sqrt{13}\, \{\cos(\tfrac{1}{2}\varphi) + i \sin(\tfrac{1}{2}\varphi)\} = 3 + 2\, I$
   5. and so

$$z = \frac{(2-3i) \pm 3i\sqrt{(5+12i)}}{2} = \frac{(2-3i) \pm 3i(3+2i)}{2} = \frac{(2-3i) \pm (9i-6)}{2}$$

   6. which gives the two roots
      a. $4 - 6i$
      b. $-2 + 3i$