Simultaneous Equations

Introduction

In general, a set of simultaneous equations link a set of unknown variables (x,y,x,..) through a set of equations. In general if there are the same number of variables as there are equations, then the equations can be solved to find the variables.

Linear equations with constant coefficients

The most common case of simultaneous equations are those in which the variables only occur as themselves, with constant coefficients. For example, consider the following set of equations

$$3x + 5y + z = 7$$
$$2x - 4y - 9z = 36$$
$$x - 9y - 2z = 1$$

With three equations in three variables there will in general be a solution of these equations. (Although, see the condition below.)

Solving linear equations with constant coefficients

There a three basic methods which can be used to solve these equations

- Substitution methods which progressively reduce the number of equations by one, until only one equation in one variable is left. This allows this variable to be found, and then by working backwards the other variables are found. This method works well when only two variables and two equations are involved, but becomes increasingly tedious as the number of variables and equations increases.
- By the use of determinants. Again this is an efficient method when there are only a few variables and equations, but as this number increases so does the number of determinants to be evaluated, and the size of the determinants involved.
- By the use of matrices. This method is efficient, given an efficient

---

Using Maple

The basic command for solving simultaneous equations using Maple is 'solve'. It the syntax

>solve({list of equations},{list of variable});

Taking the equations given in the example these are introduced into Maple as

>eqn1:=3*x + 5*y +z = 7;
>eqn2:=2*x - 4*y -9*z = 36;
>eqn3:=x - 9*y - 2*z = 1;

Maple will then solve these equations using the command

>solve({eqn1,eqn2,eqn3},{x,y,z});

with the result

$$\{x = 2, y = 1, z = -4\}$$

method for finding the inverse of a matrix.

Restrictions on solving linear equations with constant coefficients

If we write the above set of equations in matrix form ($\mathbf{AX} = \mathbf{B}$)

$$\begin{bmatrix} 3 & 5 & 1 \\ 2 & -4 & -9 \\ 1 & -9 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 36 \\ 1 \end{bmatrix}$$

then the solution is $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$. This requires that the inverse of the matrix $\mathbf{A}$ actually exists, which in turn requires that the determinant of $\mathbf{A}$ be non-zero. In this case the condition is satisfied, direct calculation shows that det($\mathbf{A}$) = -258.

However, consider the equations

$$3x + 5y + z = 7$$
$$2x - 4y - 9z = 36$$
$$8x + 6y - 7z = 1$$

Defining a matrix $\mathbf{A}$ by

$$A = \begin{bmatrix} 3 & 5 & 1 \\ 2 & -4 & -9 \\ 8 & 6 & -7 \end{bmatrix}$$

it can be readily shown that det($\mathbf{A}$) = 0. In this case there are two possibilities, the equations are inconsistent (which is actually the case here) or one of the equations can be derived from the others and no unique solution exists.

Maple and unsolvable equations

If you issue the commands

>eqn1:=3*x + 5*y +z = 7;
>eqn2:=2*x - 4*y -9*z = 36;
>eqn3:=8*x +6*y - 7*z = 1;
>solve({eqn1,eqn2,eqn3},{x,y,z});

then Maple will not respond with an answer. It cannot, the equations are inconsistent, there is no solution. However modifying eqn3 to read

>eqn1:=3*x + 5*y +z = 7;
>eqn2:=2*x - 4*y -9*z = 36;
>eqn3:=8*x +6*y - 7*z = 50;
>solve({eqn1,eqn2,eqn3},{x,y,z});

gives the result

{x = -41/29*y+99/29, z = -22/29*y-94/29, y = y}

The equations are not independent, and there is no unique solution. Maple has chosen to keep y as a variable, and to solve for x and z in terms of y.

Nonlinear equations

Maple can be used to solve nonlinear equations. As an example, consider the equations

$$\sin^3(x) + \sin(y) + z = 9$$
$$2\cos(x) - 4\cos(y) - 9z = 36$$
$$\sin(x) - 9\cos(y) - 2z = 1$$

Solving these equations by hand is a daunting task. However, Maple will find a solution to these equations.

Maple and nonlinear equations

To get the solution to the equations in the example, use the commands

```
>eqn1:=sin(x)^3+sin(y)+z=9;
>eqn2:=2*cos(x) - 4*cos(y) -9*z = 36;
> eqn3:=      sin(x) -9*cos(y) - 2*z = 1;
> res:=solve({eqn1,eqn2,eqn3},{x,y,z}):
> evalf(res[1]);evalf(res[2]);evalf(res[3]);
```

and the result is

$$x = -2.651396346 + 1.609135585\ I$$
$$z = -4.881953698 + .3944979774\ I$$
$$y = -.7192266079 - .4723039785\ I$$

Note - the algebraic output of the 'solve' routine has been assigned to the variable array 'res', but has been otherwise suppressed by terminating the line with a colon. Otherwise the response from Maple is unwieldy. The next line gives the numerical value of each of the three elements of the array 'res'.