

## Matrices in Maple

### Introduction

Maple has the capability to perform all of the matrix algebra that you might encounter. Note that many of the commands for matrix operations are contained in the library 'linalg'. It is advisable to load this library at the beginning of the worksheet. (Use with(linalg)):

### Defining matrices and matrix elements

The Maple command for defining a matrix is 'matrix'. It takes as its arguments the number of rows, then the number of columns, plus (optionally ) the matrix elements

- `A:=matrix(2,3);` - defines a matrix with two rows and three columns, and assigns it to the variable A. At the time of definition the matrix elements are undefined.
- `A:=matrix(2,3,[a,b,c,d,e,f]);` - defines the matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \quad (1)$$

and assigns it to the variable A.

- `A:=matrix(2,3,[1,7,4,-9]);` - defines the matrix

$$\begin{bmatrix} 1 & 7 & 4 \\ -9 & A_{2,2} & A_{2,3} \end{bmatrix} \quad (2)$$

and assigns it to the variable A. Four of the elements are defined, the others are not.

### Viewing a matrix

To see the current state of the matrix **A** use the command 'print(A);'

### Matrix arithmetic

All matrix operations are performed using the command 'evalm'

- `evalm(A + B);` - adds two matrices together, element by element. Note- the matrices must have the same size, although some elements can be undefined.
- `evalm(A - B);` - takes the difference between two matrices, element by element. Note- the matrices must have the same size, although some elements can be undefined.
- `evalm(A &* B);` - multiplies two matrices together. Notes - the order of multiplication is important. `A&*B` is not the same as `B&*A`. Also the number of columns of the matrix **A** must match the number of rows of the matrix **B**.

- `evalm(A / B)`; - calculates the ratio  $\mathbf{A/B}$ , equivalent to  $\mathbf{A}*(1/\mathbf{B})$ . Since the inverse of a matrix is only defined for a square matrix,  $\mathbf{B}$  must be square. Also the number of columns of the matrix  $\mathbf{A}$  must match the size (rows or columns) of  $\mathbf{B}$ .
- `evalm(A^N)`; - calculates the matrix  $\mathbf{A}$  raised to the Nth power, that is  $\mathbf{A}^N = \mathbf{A}\mathbf{A}\mathbf{A}\mathbf{A}\dots\mathbf{A}$ . From the rule for multiplying matrices,  $\mathbf{A}$  must be square.

### Determinants and inverse of a matrix

After defining a *square* matrix

$$\mathbf{C} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

- `det(C)`; - gives the determinant of C, in this case  $|C| = -16$
- `inverse(C)`; - gives the inverse of C

$$\mathbf{C}^{-1} = \begin{bmatrix} 0 & -1/4 & 1/2 \\ -1/4 & 1/2 & -1/4 \\ 1/2 & -1/4 & 0 \end{bmatrix}$$

### Eigenvalues and Eignenvectors

Given a *square* matrix  $\mathbf{A}$ , the equation  $\mathbf{AX} = \lambda\mathbf{X}$  defines and eigenvector  $\mathbf{X}$  as a column matrix (or vector) whose number of rows matches the size of  $\mathbf{A}$ , and an eigenvalue  $\lambda$ . In general there are as many solutions for  $\lambda$  and  $\mathbf{X}$  as there are rows (or columns) of  $\mathbf{A}$ , although some may be duplicate.

If the matrix

$$\mathbf{C} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

is defined using `C:=matrix(2,2,[1,2,2,4])`; then

- `eigenvalues(C)`; - will list the two possible values of the eigenvalue  $\lambda$  which satisfies the defining equation, that is 0 and 5
- `eigenvectors(C)`; - will list the eigenvalues, with their corresponding eigenvectors, and the number of times each solution is duplicated (in this case just once each)
  - 0, 1, [-2, 1]
  - 5, 1, [1, 2]