Writing the differential

In Maple the function which corresponds to the differential is ‘diff’. It takes two arguments; the function to be differentiated, and the differential variable. (If the function contains two variables, then ‘diff’ calculates the partial differential with respect to the variable listed.) For example to find the differential (with respect to x) of \((2x^2+x)e^x\) type

\[
\text{\texttt{> f:=(2*x^2+x)*exp(4*x);}}
\]

\[
\text{\texttt{> diff(f,x);}}
\]

\[
\text{\texttt{> simplify(%);}}
\]

The last line cleans up the output a little by collecting terms together. The ‘collect’ function could have been used as an alternative.

Sometimes, especially in differential equations, you will need to write the differential \(dy/dx\) when \(y\) is not explicitly known. Not that Maple treats \(x\) and \(y\) as independent variables, and \(\text{diff}(y,x)\) returns 0. As far as Maple is concerned \(y\) does not depend on \(x\). You can cure this problem by telling Maple that \(y\) is to be considered as dependent on \(x\) by writing \(y(x)\) instead of just \(y\). In Maple \(dy/dx\) is written as

\[
\text{\texttt{> diff(y(x),x);}}
\]

You can repeat the ‘diff’ function as many times as needed to calculate higher order derivatives. For example

- the third derivative of \((2x^2+x)e^x\) is written as
  \[
  \text{\texttt{> diff(diff(diff(f,x),x),x);}}
  \]
  with the result \((128x^2 + 256x + 96) e^x\)
- the second derivative \(d^2y/dx^2\) is written as
  \[
  \text{\texttt{> diff(diff(y(x),x),x);}}
  \]

Writing a differential equation

Any differential equation can be constructed by using the ‘diff’ command to represent the differential. For example to define the differential equation

\[
\frac{d^3x}{dt^3} - 7 \frac{d^2x}{dt^2} + 14 \frac{dx}{dt} - 8 = 40 \sin(2t)
\]

type

\[
\text{\texttt{> eqn:=diff(diff(diff(x(t),t),t),t)-7*diff(diff(x(t),t),t)+14*diff(x(t),t)-8*x(t)=40*sin(2*t);}}
\]

Note that \(x\) must be written as \(x(t)\) all the way through the equation, not just in the differentials.
Solving a differential equation

The function to solve a differential equation is ‘dsolve’. To solve the above equation (note that I have called the equation ‘eqn’ to save entering it again) type

\[ \text{dsolve(eqn,x(t))}; \]

which tells Maple to solve the equation that I earlier defined as ‘eqn’ for the variable x(t). The result is

\[ x(t) = \sin(2t) - \cos(2t) + c_1 e^t + c_2 e^{2t} + c_3 e^{4t} \]