Coordinate systems

- Coordinate systems
  - Cartesian Coordinates
    - (x,y,z)
  - Cylindrical Polar Coordinates
    - (η,φ,z)
  - Spherical Polar Coordinates
    - (r,θ,φ)

Coordinate Transformations

- Cartesian ⇄ Cylindrical Polar Coordinates
  - $x = \eta \cos\varphi$
  - $y = \eta \sin\varphi$
  - $z = z$
  - $r^2 = x^2 + y^2$
  - $\tan\varphi = y/x$
  - $\cos\varphi = x/\eta$
  - $\sin\varphi = y/\eta$
- Cartesian ⇄ Spherical Polar Coordinates
  - $x = r \sin\theta \cos\varphi$
  - $y = r \sin\theta \sin\varphi$
  - $z = r \cos\theta$
  - $r^2 = x^2 + y^2 + z^2$
  - $\cos\theta = z/r$
  - $\tan\varphi = y/x$
  - $\cos\varphi = x/\eta$
  - $\sin\varphi = y/\eta$

Unit Vectors

- Cartesian coordinates
  - ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$)
  - $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
- Cylindrical Polar Coordinates
  - ($\boldsymbol{\eta}$, $\boldsymbol{\varphi}$, $\mathbf{k}$)
  - $\mathbf{r} = \eta\boldsymbol{\eta} + z\mathbf{k}$
  - $\boldsymbol{\eta} = \cos\varphi\ \mathbf{i} + \sin\varphi\ \mathbf{j}$
  - $\boldsymbol{\varphi} = -\sin\varphi\ \mathbf{i} + \cos\varphi\ \mathbf{j}$
- Spherical Polar Coordinates
  - ($\mathbf{r}$, $\boldsymbol{\theta}$, $\boldsymbol{\varphi}$)
  - $\mathbf{r} = r\mathbf{r}$
  - $\mathbf{r} = \sin\theta \cos\varphi\ \mathbf{i} + \sin\theta \sin\varphi\ \mathbf{j} + \cos\theta\ \mathbf{k}$
  - $\boldsymbol{\theta} = \cos\theta \cos\varphi\ \mathbf{i} + \cos\theta \sin\varphi\ \mathbf{j} - \sin\theta\ \mathbf{k}$
  - $\boldsymbol{\varphi} = -\sin\varphi\ \mathbf{i} + \cos\varphi\ \mathbf{j}$

Area and Volume Elements

- Cartesian coordinates
  - Area element parallel to xy plane, $d\mathbf{S} = dx\, dy\, \mathbf{k}$
  - Area element parallel to yz plane, $d\mathbf{S} = dy\, dz\, \mathbf{i}$
  - Area element parallel to xz plane, $d\mathbf{S} = dx\, dz\, \mathbf{j}$
  - Volume element, $dv = dx\, dy\, dz$
- Cylindrical Polar Coordinates
  - Area element parallel to xy plane, $d\mathbf{S} = \pm\, \eta\, d\eta\, d\varphi\, \mathbf{k}$
  - Area element of curved surface, $d\mathbf{S} = \eta\, d\varphi\, dz\, \boldsymbol{\eta}$
  - Volume element, $dv = \eta\, dr\, d\varphi\, dz$
- Spherical Polar Coordinates
  - Area element, $d\mathbf{S} = r^2 \sin\theta\, d\theta\, d\varphi\, \mathbf{r}$
  - Volume element, $dv = r^2 \sin\theta\, dr\, d\theta\, d\varphi$