Phys 3010 Mathematical Physics I
Fall 2018
Final Exam - due by noon Friday December 14th 2018

Note: Questions should be solved by hand, except that you may use Maple to evaluate complicated integrals, and to check your answers.

1. Suppose we take the curve $y = a^2 - x^2$ and rotate it once about the x axis. If there is a charge density $\rho = \rho_o\, xy/a^2$, what is the total charge inside the volume that is formed?

2. Five waves of equal frequency (ω) are made to interfere. The amplitudes of the waves are equal to A, A, 2A, 4A,and 5A. Their phases angles (relative to that of the first wave) are 0, $\pi/3$, $\pi/4$, $\pi/6$, and $2\pi/3$. Find the amplitude, frequency, and phase angle of the resulting wave.

3. ***Without resorting to any specific coordinate system***, find the following
   a. div (**m** x (**r** x **m**)), where **m** is a constant vector
   b. curl (**m** x (**r** x **m**)), where **m** is a constant vector

4. Write down the binomial series for $y = 1/(1+x)$, then integrate and use the result to show that

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} = \ln(2)$$

5. Find the work done ($W = \int \mathbf{F}.d\mathbf{r}$) by the force $\mathbf{F} = xy\mathbf{i} + y^2\mathbf{j}$ from the point (4,0,0) to the point (0,0,0)
   a. along the straight line between these two points
   b. along the semicircle $(x-2)^2 + y^2 = 4$ with $y>0$ and $z=0$.
   c. Is the force conservative?

6. Find the series solution for the differential equation

$$\left(x^2+1\right)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 2x$$

   subject to the conditions that $y=1$ and $dy/dx=4$ both at $x=0$.

7. Solve the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 20x^3e^{x/2}$$

   given that $y=4$ and $dy/dx=6$ both when $x=0$.

8. Find the moment of inertia of a hollow sphere of radius a and centred on the origin if it is rotated about an axis parallel to the x axis through the point (0,½a,½a).
   Show that your result is consistent with the Parallel Axis Theorem. (Moment of inertia of a hollow sphere about any axis through its centre is $\frac{2}{3}ma^2$)