Phys 3010 Mathematical Physics
Assignment 9

---

1. Find the gradient of
   a. $(2x+y)(x-z)$ $(4x+y-2z)\mathbf{i} + (x-z)\mathbf{j} - (2x+y)\mathbf{k}$
   b. $1/r^n$ (spherical coordinates) – $\frac{-n}{r^{n+1}}\hat{\boldsymbol{r}} = \frac{-n}{r^{n+2}}\boldsymbol{r}$
   c. $r^3 \sin\theta \cos^2\varphi$ (spherical coordinates) $3r^2\sin\theta\cos^2\phi\,\hat{\boldsymbol{r}} + r^2\cos\theta\cos^2\phi\,\hat{\boldsymbol{\theta}} - 2r^2\sin\phi\cos\phi\,\hat{\boldsymbol{\phi}}$

2. Find the divergence of
   a. $x^2\mathbf{i} + 2yz\mathbf{j} + 2yz\mathbf{k}$ $2(x+y+z)$
   b. $r^n\,\mathbf{r}$ (spherical coordinates) $(n+3)\,r^n$
   c. $r^3 \sin\theta \cos^2\varphi \;\; \hat{\boldsymbol{r}}$ (spherical coordinates) $5r^2\sin\theta\cos^2\varphi$

3. For each of the vectors in the previous question find its curl.
   a. $\text{curl}(x^2\mathbf{i} + 2yz\mathbf{j} + 2yz\mathbf{k}) = 2(z-y)\,\mathbf{i}$
   b. $\text{curl}(r^n\,\mathbf{r}) = 0$
   c. $\text{curl}(r^3 \sin\theta \cos^2\varphi \;\; \hat{\boldsymbol{r}}) = 2\,r^2 \sin\varphi \cos\varphi \;\; \hat{\boldsymbol{\theta}} - r^2 \cos\theta \cos^2\varphi \;\; \hat{\boldsymbol{\phi}}$

4. Show that in cylindrical polar coordinates $\text{div}(\mathbf{r}) = 3$. A cylinder of length 0.7 m and radius 0.25 m has a hole bored out along half its length, with the axis of the hole parallel to the axis of the original cylinder but through the point (0.1 m, 0, 0). The radius of the hole is 0.125 m. What is the value of $\oint \mathbf{r}.d\mathbf{A}$ integrated over the surface of what remains of the cylinder?

$$\mathbf{r} = r\,\hat{\boldsymbol{r}} + z\,\boldsymbol{k}$$

$$\text{div}(\mathbf{r}) = \frac{1}{r}\frac{\partial}{\partial r}(rr) + \frac{\partial}{\partial z}x = 2+1 = 3$$

$$\oint \boldsymbol{r}.d\boldsymbol{A} = \int \text{div}\;\mathbf{r}\,dv = \int 3\,dv = 3V$$

$$\oint \boldsymbol{r}.d\boldsymbol{A} = 3\left(\pi R^2 L^2 - \pi r^2 l^2\right)$$

$$\oint \boldsymbol{r}.d\boldsymbol{A} = 3\,\pi\left(0.25^2\,0.7 - 0.125^2\,0.35\right) = 0.36\,m^3$$