

## Phys 3010 Mathematical Physics

### Assignment 8

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1. Show that the moment of inertia of a uniform hollow sphere rotating about an axis through its center is  $\frac{2}{3} Ma^2$ . (Hint for  $dm$  use  $\sigma dA$ , which  $\sigma$  a constant.)
2. A uniform rod of length  $L$  has a mass  $M$ . If it rotates about an axis perpendicular to the rod and through one end, show that the moment of inertia is  $\frac{1}{3} ML^2$ .
3. Calculate the moment of inertia of the rod in the previous problem if it rotates about an axis perpendicular to the rod and through its centre?
4. Repeat problem 2 if the rod is not uniform, but has a line density  $\lambda = \lambda_0 |x|/L$  where  $x$  is the distance from the center. (In this problem  $x=0$ , the origin, is at the center of the rod. The rod lies between  $x = \frac{1}{2}L$  and  $x = -\frac{1}{2}L$ .)
5. Define the volume defined by that portion of a sphere of radius 1 m and which had  $\theta \leq 30^\circ$ .
  - a. If the volume is filled with material of constant mass density, find the centre of mass.
  - b. If the volume is instead filled with a charge whose density is  $\rho = \sin(\theta + \varphi)$ , find the total charge.
6. Repeat the previous problem if the region above  $z = \cos(30^\circ)$  is removed. (That is, the top is cut off). Hint: there are two quite different approaches to this problem. They are both correct and yield the same answers.