

## Phys 3010 Mathematical Physics

### Assignment 7

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1. Find the moment of inertia of a solid sphere of radius  $a$  about any axis passing through its centre if the density of the sphere varies as  $\rho = \rho_0 r^n/a^n$ .
  - a. A value of  $n=0$  corresponds to a uniform density  $\rho = \rho_0$ , for which you already know the moment of inertia from General Physics I,  $I = \frac{2}{5}ma^2$ . Show that your answer reduces to this value for  $n=0$ .
  - b. A value of  $n \rightarrow \infty$  corresponds to a mass which is concentrated on the surface of the sphere, for which you already know the moment of inertia from General Physics I,  $I = \frac{2}{3}ma^2$ . Show that your answer reduces to this value as  $n \rightarrow \infty$ .  
(Note: testing a general result against known specific cases is a useful technique for spotting errors.)
2. A sphere of radius  $a$  is cut into eight by the three coordinate planes. If there is a uniform mass density, then for the eighth which has  $x, y,$  and  $z$  all  $>0$ , find the centre of mass. (Hint: first think about where you might expect the centre of mass to be.)
3. The shape in the previous problem is made to rotate about the  $z$  axis. Find its moment of inertia.
4. The shape in the previous two problems also contain a non-uniform charge density  $\rho = \rho_0 xy/a^2$ . Find the total charge.