Phys 3010 Mathematical Physics
Assignment 6

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1. Find the moment of inertia of a solid sphere of radius a about any axis passing through its centre if the density of the sphere varies as $\rho = \rho_o r^n/a^n$.
   a. A value of n=0 corresponds to a uniform density $\rho = \rho_o$, for which you already know the moment of inertia from General Physics I, $I = \frac{2}{5}ma^2$. Show that your answer reduces to this value for n=0.
   b. A value of $n \to \infty$ corresponds to a mass which is concentrated on the surface of the sphere, for which you already know the moment of inertia from General Physics I, $I = \frac{2}{3}ma^2$. Show that your answer reduces to this value as $n \to \infty$.
   (Note: testing a general result against known specific cases is a useful technique for spotting errors.)

2. A sphere of radius a is cut into eight by the three coordinate planes. It is made to rotate about the z axis. If there is a uniform mass density, then for the eighth which has x, y, and z all >0, find its moment of inertia.

3. The shape in the previous two problems also contain a non-uniform charge density $\rho = \rho_o xy/a^2$. Find the total charge.

4. If the electric field is given by $\mathbf{E} = \sin(z)\,\mathbf{j}$, calculate the electric flux crossing the triangle defined by the points (x,y,x) = (0,0,0), (4,0,0), and (0,0,3). (Hint: draw a diagram. Note also that your limits are not going to be constant.)

5. Given a force $\mathbf{F} = (xy+2)\mathbf{i} + (x+z)\mathbf{j} + x^2\mathbf{k}$, find the work done in moving from the origin to the point (4,4,1)
   a. Along the path $(0,0,0) \to (4,0,0) \to (4,4,0) \to (4,4,1)$
   b. Along the path $(0,0,0) \to (0,4,0) \to (0,4,1) \to (4,4,1)$
   c. Along the straight line between the initial and final points.
   d. Along the curved path defined by the pair of equations $5x = y^2+y = 20z$.