Phys 3010 Mathematical Physics
Assignment 17

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1. The asymmetric square wave f(t)=1 for 0≤t≤¼T, f(t)=0 for ¼T≤t≤¾T, and f(t)=-1 for ¾T≤t≤T.
   a. $a_n = \frac{\sin(\frac{n\pi}{2}) + \sin\left(\frac{3n\pi}{2}\right)}{n\pi} = \frac{2\sin\left(\frac{4n\pi}{2}\right)\sin\left(\frac{2n\pi}{2}\right)}{n\pi} = 0$ for any integer value of n
   b. $b_n = \frac{2 - \cos(\frac{n\pi}{2}) - \cos\left(\frac{3n\pi}{2}\right)}{n\pi} = \frac{2}{n\pi}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]$ if n is even, or = $\frac{2}{n\pi}$ if n is odd
   c. $a_0 = 0$

2. The triangle wave, which rises linearly to f(½T) = 1 and then falls linearly to f(T)=0.
   a. $f(t) = \frac{2t}{T}$ *if* $t < \frac{T}{2}$, $f(t) = 2 - 2\frac{t}{T}$ *if* $t > \frac{T}{2}$
   b. $a_n = \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right] = 0$ *if n is even*, $= -\frac{4}{n^2\pi^2}$ *if n is odd*
   c. $b_n = 0$
   d. $a_0 = ½$

3. The sawtooth wave, which rises linearly from f(0)=0 to f(T)=1.
   a. $f(t) = \frac{t}{T}$
   b. $a_n = 0$
   c. $b_n = -\frac{1}{n\pi}$
   d. $a_0 = ½$

4. The wave f(t) = sin(2πt/T)*cos^3(6πt/T). Note: Maple will find four of these integrals to be undefined using a general formula, as there is the possibility that the denominator is zero. Set up each one of these separately.
   a. The general expression for both a_n and b_n contains the denominator $409600 - 138496n^2 + 9744n^4 - 184n^6 + n^8$
   b. If this denominator is not zero then a_n and b_n are both zero
   c. The denominator is zero if n is 2, 4, 8, or 10. these terms must be evaluated separately. (There are also some negative roots which you can ignore as n must be positive.)
   d. $b_2 = -⅜$, $a_2 = 0$
   e. $b_4 = +⅜$, $a_4 = 0$
   f. $b_8 = -⅛$, $a_8 = 0$
   g. $b_{10} = +⅛$, $a_{10} = 0$
   h. $a_0 = 0$
   i. f(t) is therefore a four term sum

$$f(t) = -\frac{3}{8}\sin\left(\frac{4\pi t}{T}\right) + \frac{3}{8}\sin\left(\frac{8\pi t}{T}\right) - \frac{1}{8}\sin\left(\frac{16\pi t}{T}\right) + \frac{1}{8}\sin\left(\frac{20\pi t}{T}\right)$$