**Phys 3010 Mathematical Physics**
**Assignment 16**

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Please work out the following by hand. You can check your answers using Maple if you like. You can also use the results from class, that is the series we had for

- sin(x)
- cos(x)
- $e^x$
- $(1+x)^n$

1. Find the Maclaurin/Binomial series for 1 / (1+x). $1 - x + x^2 - x^3 + \ldots = \sum (-1)^n x^n$

2. Find the Maclaurin/Binomial series for ln(1+x). (Hint, use the result of the previous question)

   a. By direct integration $\ln(1+x) = C + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots = \sum (-1)^n \frac{x^n}{n}$ where C I sna integration constant.
   b. Put x= 0 to show that C = ln(1) = 0

3. Find the Maclaurin series for $\tan^{-1}(x)$ [hint: use the result of question 1, but replace x with $x^2$], and use it to find the limiting value of f(x) as x→0 if

$$f(x) = \frac{\tan^{-1}(x) - \sin(x)}{x^3}$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \ldots$$

$$\tan^{-1}(x) - \sin(x) = -\frac{x^3}{6} + \frac{23}{120}x^5 - \frac{719}{5040}x^7 + \ldots$$

As x→0 $\tan^{-1}(x)$-sin(x) → $-x^3/6$, and so f(x) → -1/6.

4. A quadrupole can be defined as the sum of the charges +Q at the point a**k**, -2Q at the origin, and +Q at the point -a**k**, Find the electrostatic potential at the point P, given that r≫a. (Note: there is more cancellation in this problem than there was in class when we looked at the dipole. That means you will need to keep more terms in the Binomial expansions.)