

Phys 3010 Mathematical Physics

Assignment 13 Answers

1. $\frac{dx}{dt} + 5x = 4e^{-5t}$

a. The CF has a term in e^{-5t} , as does the RHS of the differential equation. The PI must therefore have a term te^{-5t} .

b. $x = Ae^{-5t} + 4te^{-5t}$

2. $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 100\sin(2t)$

a. The equation $m^2 + 8m + 16 = 0$ has a repeated root. The CF must therefore have terms in e^{mt} and te^{mt} .

b. $x = Ae^{-4t} + Bte^{-4t} - 4\cos(2t) + 3\sin(2t)$

3. $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 5x = 4e^t$

a. The PI has a term in e^t , as does the RHS of the differential equation. The PI must therefore have a term te^t .

b. $x = Ae^t + Be^{5t} - te^t$

4. $\frac{d^4x}{dt^4} - 7\frac{d^3x}{dt^3} + 9\frac{d^2x}{dt^2} + 27\frac{dx}{dt} - 54x = 30e^{3t}$

a. The equation $m^4 - 7m^3 + 9m^2 + 27m - 54 = 0$ has four roots, -2 and +3 three times. With one root repeated three times this requires terms in e^{3t} , te^{3t} , and t^2e^{3t} .

b. The RHS of the differential equation also contains e^{3t} . Since the CF has terms in e^{3t} , te^{3t} , and t^2e^{3t} . The PI must have a term t^3e^{3t} .

c. $x = Ae^{-2t} + Be^{3t} + Cte^{3t} + Dt^2e^{3t} + t^3e^{3t}$