Phys 3010 Mathematical Physics
Assignment 11

This assignment contains some very useful relations which you are likely to encounter in subsequent classes. We have already met one in an earlier assignment, div**r** = 3.

Preliminaries

1. Show that
   a. curl(r^n $\hat{\mathbf{r}}$) = 0 for any value of n
   b. grad(r^n) = nr^{n-1} $\hat{\mathbf{r}}$ = $nr^{n-2}\mathbf{r}$
   c. (**a.∇**)**r** = **a**, for any vector **a**, even if it is not constant.

Product rule questions

The rest of the questions should be answered ***without resorting to any particular coordinate system***. Instead use the above identities, plus the product rules on the handout from class.

2. If **r** is the position vector, and **m** is any constant vector, find the following:
   a. div r^n **r**. Most importantly what is the result if n = -2? (This corresponds to a field or force which obeys the inverse square law, such a gravity or the electrostatic force.)
   b. div (**m**x**r**)
   c. curl (**m**x**r**)
   d. grad (**m.r**/r^3) [There are three possible starting points. Start with the expression for grad(fg), it is the easiest. This is an important relationship for dipoles, both electric and magnetic. We will use it later.]

3. If f**a** = grad(g), show that **a**.(curl **a**) = 0, regardless of the functions f and g, except for f=0. (Hint: since the final expression requires you to know curl **a**, start by taking the curl of both sides of the original expression.)