1. Show that
   a. curl($r^{n\wedge}r$) = 0 for any value of n
      i. The expression for the curl(**Q**) of a vector contains 6 derivatives. The vector r^n**r** has only a radial component, and so the four derivatives involving Q_θ and Q_φ are automatically 0. The remaining two are the derivatives of $Q_r = r^n$ with respect to θ and φ,which are also 0.
   b. grad(r^n) = nr^{n-2}**r**
      i. $$grad(r^n)=\frac{\partial r^n}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial r^n}{\partial \theta}\hat{\boldsymbol{\theta}}+\frac{1}{r\sin\theta}\frac{\partial r^n}{\partial \phi}\hat{\boldsymbol{\phi}}=nr^{n-1}\hat{\mathbf{r}}=nr^{n-2}\mathbf{r}$$
   c. (**a**.∇)**r** = **a**, for any vector a, even if it is not constant.
      i. $$(\mathbf{a}.\nabla)\mathbf{r}=\left(a_x\frac{\partial}{\partial x}+a_y\frac{\partial}{\partial y}+a_z\frac{\partial}{\partial z}\right)(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})=a_x\mathbf{i}+a_y\mathbf{j}+a_z\mathbf{k}=\mathbf{a}$$

Product rule questions

2. The rest of the questions should be answered without resorting to any particular coordinate system. Instead use the above identities, plus the product rules on the handout from class.
   a. If r is the position vector, and m is any constant vector, find the following:
   b. div rn r. Most importantly what is the result if n = -2? (This corresponds to a field or force which obeys the inverse square law, such a gravity or the electrostatic force.)
   c. div (**m**x**r**)
      i. div (**mxr**) = **r** x curl(**m**) **–** **m**xcurl(**r**)
      ii. curl(**r**) = 0 see question 1, part a with n = 1
      iii. curl(**m**) = 0 since **m** is a constant.
      iv. div (**mxr**) = 0
   d. curl (mxr)
      i. curl (**mxr**) = div(**r**) **m** - div(**m**) **r** + (**r**.∇)**m** - (**m**.∇)**r**
      ii. div(**r**) = 3
      iii. (**r**.∇)**m** = **m**, see question 1, part c
      iv. div(**m**) and (**r**.∇)**m** are both 0, since **m** is a constant.
      v. curl (**mxr**) = 3 **m** – 0 + 0 – **m** = 2 **m**
   e. grad (**m**.**r**/r^3)
      i. grad (**m**.**r**/r^3) = grad (**m**.**r***1/r^3) = 1/r^3 grad(**m**.**r**) + **m**.**r** grad(1/r^3)
      ii. grad(**m**.**r**) = **m** x curl(**r**) + **r** x curl(**m**) + (**r**.∇)**m** + (**m**.∇)**r**
      iii. grad(**m**.**r**) = 0 + 0 + 0 + (**m**.∇)**r** = **m**
      iv. grad(1/r^3) = - (3/r^5) **r**
      v. grad (**m**.**r**/r^3) = 1/r^3 **m** - **m**.**r** (1/r^5) **r** = (1/r^5) [r^2**m** – 3(**m**.**r**)**r**]
   f. If fa = grad(g), show that a.(curl a) = 0, regardless of the functions f and g, except for f=0.
      i. curl(f**a**) = curl(grad(g)
      ii. f curl(**a**) + grad(f) x **a** = 0
      iii. Take the dot product with **a**, f **a**.curl(**a**) + **a**.grad(f) x **a** = 0
      iv. The second term is a scalar triple product with two vectors the same, and so is zero.
      v. If f≠0 divide by f,which gives the result **a**.curl(**a**) = 0