1. $$\text{div}\,\boldsymbol{E}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 r^2 \sin\theta)+\frac{1}{r}\frac{\partial}{\partial\theta}(\sin\theta 4r^2\cos\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(r\tan\theta)$$
$$\text{div}\,\boldsymbol{E}=4r\sin\theta+4r\frac{(\cos^2\theta-\sin^2\theta)}{\sin\theta}=4r\frac{\cos^2\theta}{\sin\theta}$$
$$\int\boldsymbol{E}.d\boldsymbol{A}=\int\text{div}\,\boldsymbol{E}\,dv=\int_o^a\int_0^{\frac{\pi}{6}}\int_0^{2\pi}4r\frac{\cos^2\theta}{\sin\theta}r^2\sin\theta dr\,d\theta d\phi$$
$$\int\boldsymbol{E}.d\boldsymbol{A}=\int_0^a 4r^3dr\int_0^{\frac{\pi}{6}}\cos^2\theta d\theta\int_0^{2\pi}d\phi = a^4\ \frac{1}{2}\left[\theta+\frac{1}{2}\sin2\theta\right]_0^{\pi/6}\ 2\pi$$
$$\int\boldsymbol{E}.d\boldsymbol{A}=a^4\left[\frac{\pi}{12}+\frac{\sqrt{3}}{8}\right]2\pi$$

2. Solving in spherical polar coordinates, the solution is the same as question 1, except the limit on r is now 0 to acos(π/6)/cos(θ). The r integral now has to be done before the θ integral.
$$\int\boldsymbol{E}.d\boldsymbol{A}=\int_0^{\frac{\pi}{6}}\left[\int_0^{\frac{acos(\pi/6)}{\cos\theta}}4r^3dr\right]\left(\frac{\cos^2\theta}{\sin\theta}\right)\sin\theta d\theta\int_0^{2\pi}d\phi=\frac{3\sqrt{3}}{8}\pi a^4$$
Converting to cylindrical polar coordinates
$$\text{div}\,\boldsymbol{E}=4r\frac{\cos^2\theta}{\sin\theta}=4\frac{r^2\cos^2\theta}{r\sin\theta}=4\frac{z^2}{s}\qquad(\textit{using s for the radial coordinate})$$
$$\int\boldsymbol{E}.d\boldsymbol{A}=\int\text{div}\,\boldsymbol{E}\,dv=\int_0^{a\cos(\pi/6)}\left[\int_0^{z\tan(\pi/6)}4\frac{z^2}{s}s\,ds\right]dz\int_0^{2\pi}d\phi=\frac{3\sqrt{3}}{8}\pi a^4$$

3. The line integral has to be split into 3
   a. From (0,0,0) to (0,1,0)
      i. x=0, z = 0, d**r** = dy **j**
      ii. a = 6**i** + 0**j** + 3y**k**
      iii. ∫**a**.d**r** = ∫(6**i** + 0**j** + 3y**k**).(dy **j**) = 0
   b. From (0,1,0) to (0,0,2)
      i. x = 0, z = 2-2y, d**r** = dy **j** + dz **k**= dy **j** – 2dy **k**
      ii. a = 6**i** + $y(2-2y)^2$**j** + (y+2)**k**
      iii. ∫**a**.d**r** = ∫(6**i** + $y(2-2y)^2$**j** + (y+2)**k**).(dy **j** – 2dy **k**)
      iv. ∫a.dr = $\int_1^0 (y(2-2y)^2 -2(y+2))dy$ = 14/3
   c. From (0,0,2) to (0,0,0)
      i. x = 0, y = 0, d**r** = dz **k**
      ii. a = 6**i** +0**j** + z**k**
      iii. ∫**a**.d**r** = ∫(6**i** + 0**j** + z**k**).(dz **k**) = $\int_2^0 z\,dz$ = -2
   d. ∮**a**.d**r** = 0 + 14/3 – 2 = 8/3

4. $\rho = \varepsilon_o\,\text{div}\mathbf{E} = \varepsilon_o\,(1/r^2)\,\partial/\partial r(r^2 r^3) = 5\varepsilon_o r^2 = 5\varepsilon_o\,(s^2+z^2)$ in cylindrical polar coordinates
   $Q = \int \rho\,dv = \int_0^a \int_0^{2\pi} \int_0^L 5\varepsilon_o\,(s^2+z^2)\,s\,ds\,d\varphi\,dz = \frac{1}{2}\pi a^4 L + \frac{1}{3}\pi a^2 L^3$

5. $$\rho=\epsilon_o\,\text{div}\,\boldsymbol{E}=\epsilon_o\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2r^n)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\ r^n\cos\theta)\right]$$
$$\rho=\epsilon_o\left[(n+2)r^{n-1}+r^{n-1}(\cos^2\theta-\sin^2\theta)\right]=\epsilon_o\left[(n+2)r^{n-1}+r^{n-1}\cos2\theta\right]$$

$$Q=\int \rho dv=\epsilon_o(n+2)\int_0^a r^{n-1}r^2 dr\int_o^{\pi} \sin(\theta) d\theta \int_0^{2pi} d\phi+\epsilon_o\int_0^a r^{n-1}r^2 dr\int_o^{\pi} \cos(2\theta)\sin(\theta) d\theta\int_0^{2pi} d\phi$$

$$Q=\epsilon_o(n+2)\frac{a^{n+2}}{n+2}(2)(2\pi) \;+\; \epsilon_o\frac{a^{n+2}}{n+2}(0)(2\pi)=4\pi\epsilon_o a^{n+2}$$