

Phys 3010 Mathematical Physics

Assignment 5

This assignment relies on the two expressions

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \text{and} \quad \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

1. Multiply the expressions together to show that $\sin(A)\cos(A) = \frac{1}{2} \sin(2A)$.
2. Square each and then subtract to show that $\cos^2(A) - \sin^2(A) = \cos(2A)$.
3. Take the expression for the cosine of θ to show that

$$16\cos^5(\theta) = \cos(5\theta) + 5\cos(3\theta) + 10\cos(\theta)$$

4. What is $8 \sin^4(\theta)$?
5. Find the integral

$$\int \cos^2 \theta e^{2\theta} d\theta$$

by replacing the trig function with its complex equivalent. Note that this integral is difficult to evaluate using (real) trigonometric functions but straightforward using complex notation.

6. The first result you get from the above integral looks to be complex in that it contains the quantity “i”. Simplify the result to show that it is indeed real, as it must be since you started with a real integrand.
7. Check your result against Maple to make sure that you have got the right answer. The function *evalc()* can help evaluate complex expressions.