Phys 3010 Mathematical Physics
Assignment 18

This assignment contains some very useful relations which you are likely to encounter in subsequent classes.

Preliminaries

1. Show that
   a. div$\mathbf{r}$ = 3
   b. curl(r^n $\hat{\mathbf{r}}$) = 0 for any value of n
   c. grad(r^n) = nr^{n-1} $\hat{\mathbf{r}}$ = $nr^{n-2}\mathbf{r}$
   d. ($\mathbf{a}.\nabla$)$\mathbf{r}$ = $\mathbf{a}$, for any vector $\mathbf{a}$, even if it is not constant.

Product rule questions

The rest of the questions should be answered ***without resorting to any particular coordinate system***. Instead use the results of the first question, plus the product rules on the handout from class.

2. If $\mathbf{r}$ is the position vector, $\mathbf{a}$ is any vector, and $\mathbf{m}$ is any constant vector, find the following:
   a. div $r^n\mathbf{r}$
   b. div ($\mathbf{m}$x$\mathbf{r}$)
   c. curl ($\mathbf{m}$x$\mathbf{r}$)
   d. grad ($\mathbf{m}.\mathbf{r}/r^3$) [There are three possible starting points. Start with the expression for grad(fg), it is the easiest. This is an important relationship for dipoles, both electric and magnetic. We will use it later.]

3. If f$\mathbf{a}$ = grad(g), show that $\mathbf{a}$.(curl $\mathbf{a}$) = 0, regardless of the functions f and g, except for f=0. (Hint: since the final expression requires you to know curl $\mathbf{a}$, start by taking the curl of both sides of the original expression.)