Phys 3010 Mathematical Physics
Assignment 17

You will probably want to use Maple for some of these integrals. Also, you can use Maple to check some of the vector derivatives.

1. Let $\boldsymbol{E} = r^2 \sin\theta\,\hat{\boldsymbol{r}} + 4r^2\cos\theta\,\hat{\boldsymbol{\theta}} + r\tan\theta\,\hat{\boldsymbol{\varphi}}$. Use the Divergence Theorem to find $\oint \mathbf{E}.d\mathbf{S}$ for that portion of the sphere of radius R defined by the cone whose axis is the z axis, and whose cone angle is 30°.

2. In problem 1, what difference would it make if the volume was defined by a flat top, rather than the surface of the sphere? Use the result to find the total charge inside the sphere, but with $z \geq R\cos 30^\circ$. (Hint: whereas question 1 is best solved using spherical coordinates, this question can be solved equally well using cylindrical coordinates. Either way the limits on the integrals are not constant.)

3. Let $\mathbf{a} = 6\,\mathbf{i} + yz^2\,\mathbf{j} + (3y+z)\,\mathbf{k}$. By direct integration find the value of $\oint \mathbf{a}.d\mathbf{r}$ for the triangle defined by the points (0,0,0), (0,1,0), and (0,0,2). Show that you get the same result if you use Stokes' Theorem.

4. The electric field is given by $\mathbf{E} = r^3\,\hat{\mathbf{r}}$. Find the charge density, and the total charge inside the cylinder of radius a whose axis is the z axis, of length L ($0<z<L$). (Hint: the field is given is spherical polar coordinates, but once you have calculated the charge density, switch to cylindrical polar coordinates to get the total charge.)

5. The electric field is given by $\boldsymbol{E} = r^n\hat{\boldsymbol{r}} + r^n\cos\theta\,\hat{\boldsymbol{\theta}}$. Find the charge density, and show that the total charge inside the sphere of radius a is given by $4\pi\,\varepsilon_o\,a^{n+2}$. (Assume $n \neq -2$)