

## Phys 3010 Mathematical Physics

### Assignment 16

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- Find the divergences of the following:
  - $\mathbf{A} = (2x+1)\mathbf{i} + (xy+z^2)\mathbf{j} + x/z\mathbf{k}$ .
  - $\mathbf{B} = r \cos(\varphi) \hat{r} + r \sin(\varphi) \hat{\phi}$  (Note:  $r$  is the unit vector in cylindrical polars)
  - $\mathbf{C} = r^2 \sin(\varphi) \cos(\varphi)\mathbf{i}$
  - $\mathbf{D} = r \sin(\theta) \hat{r} + r \cos(\theta) \hat{\theta} + r \sin(\theta) \cos^2(\varphi) \hat{\phi}$  (Note:  $\hat{r}$ ,  $\hat{\theta}$ , and  $\hat{\phi}$  are the unit vectors in spherical polars)
- Find the gradient of the following
  - $f = xyz$
  - $g = r \cos(2\varphi)$
  - $h = r \cos(\theta) e^{2i\varphi}$
  - $1/r$  where  $r$  is the distance from the origin. (Another useful result. Remember it.)
- For an electric field  $\mathbf{E} = r^2 \cos^2(\theta) \sin^2(\varphi) \hat{r}$  and a sphere of radius  $R$  centred on the origin, from direct integration find
  - $\oint \mathbf{E} \cdot d\mathbf{A}$
  - $\int \text{div}\mathbf{E} dv$Note: these results ought to be equal according to the Divergence Theorem.