Phys 3010 Mathematical Physics
Assignment 14

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1. Find the moment of inertia of a solid sphere of radius a about any axis passing through its centre if the density of the sphere varies as $\rho = \rho_o \, r^n/a^n$.
   a. A value of n=0 corresponds to a uniform density $\rho = \rho_o$, for which you already know the moment of inertia from General Physics I, $I = \frac{2}{5}ma^2$. Show that your answer reduces to this value for n=0.
   b. A value of $n \to \infty$ corresponds to a mass which is concentrated on the surface of the sphere, for which you already know the moment of inertia from General Physics I, $I = \frac{2}{3}ma^2$. Show that your answer reduces to this value as $n \to \infty$.
   (Note: testing a general result against known specific cases is a useful technique for spotting errors.)

2. A sphere of radius a is cut into eight by the three coordinate planes. If there is a uniform mass density, then for the eighth which has x, y, and z all >0, find the centre of mass. (Hint: first think about where you might expect the centre of mass to be.)

3. The shape in the previous problem is made to rotate about the z axis. Find its moment of inertia.

4. The shape in the previous two problems also contain a non-uniform charge density $\rho = \rho_o \, xy/a^2$. Find the total charge.

5. Find the moment of inertia of a flat uniform (density and thickness) annulus between the limits $a \le r \le b$. The axis runs through the centre of the annulus and is perpendicular to its plane. Express your final answer in terms of the mass of the annulus, a, and b.