

Phys 3010 Mathematical Physics

Assignment 13

1. The point P_2 is described in Cartesian coordinates by $(4,7,-1)$. What is the description of P_2 in spherical polars?
2. For the point in the previous question, what are the three spherical polar unit vectors, each in terms of \mathbf{i} , \mathbf{j} , and \mathbf{k} ?
3. A real world application: Suppose you want to know the distance between two points on the Earth, given that you have to go around the Earth, not in a straight line. For example what is the distance between Cape Town airport (Latitude $33^\circ 58' 10''$ S, Longitude $18^\circ 35' 50''$ E) and Toronto airport (Latitude: $43^\circ 40' 30''$ N, Longitude: $79^\circ 37' 50''$ W). Take the Earth to be a perfect sphere of radius 3970 miles, and calculate the distance.

What would be the distance if you could burrow through the Earth in order to go between the two airports in a straight line?

Hints:

- a. Think dot product of two vectors. Also, this is a lot easier to calculate in Cartesian coordinates than in spherical Polar coordinates.
- b. Think arc length along a circle, known as the great circle. It is the circle around the Earth with its centre at the centre of the Earth, and which goes through both points.
- c. Don't forget that the latitude and the polar angle are related but not the same. Latitude is measured from the Equator, but the polar angle is measured from the North Pole
- d. There are a lot of calculations to do, and so plenty of opportunities to make mistakes. Maple can be invaluable.