Phys 3010 Mathematical Physics
Assignment 11

Another differential equation that needs a series solution is

$$\frac{d^2H}{dx^2} - 2x\frac{dH}{dx} + (2E-1)H = 0$$

It is found when trying to solve Schroedinger's Equation for a harmonic oscillator. (You will likely meet this problem in Phys 4510 Quantum Mechanics I)

1. Assume a solution of the form $H = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots = \sum a_i x^i$. (The solutions are known as Hermite Polynomials.) Write down the first and second derivatives of H.

2. Substitute into the differential equation.

3. Equate terms in a given power of x to derive the recurrence relationship between a_n and a_{n+2}.

4. There is a physical requirement that the Hermite polynomials must have a finite number of terms, that is that a_{N+2} must become 0 for some value of N. Use that condition to show that the parameter E can only take on the values $E = N + ½$, where N is any non-negative integer. This condition also mandates that a given Hermite polynomial can only have either even power terms, or odd power terms, but not both.

5. Calculate the polynomial H_N for $0 \leq N \leq 4$. (You can check your answers in Maple using the function H(N,x) in Maple. You will need to load the library “orthopoly” first.)