

Phys 3010 Mathematical Physics

Assignment 10

In class we looked solving Laplace's Equation in both Cartesian (x,y) and Spherical Polar (r,θ) coordinates. There is a equivalent for for Laplace's Equation in Cylindrical Polar (r,φ) coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \varphi^2} = 0$$

1. Assume a solution of the form $V(r,\varphi) = R(r)\Phi(\varphi)$ and separate the equation into a LHS which only contains r and a RHS which only contains φ .
2. Set each side to be the same constant (k).
3. Start by solving the ordinary differential equation for $\Phi(\varphi)$. Show that the solution can written in the form $\Phi = A \cos(\lambda\varphi) + B \sin(\lambda\varphi)$, where $\lambda^2 = k$.
4. There is a physical restriction which requires that V be single valued, that is if you circle the z axis once ($\varphi \rightarrow \varphi + 2\pi$) then V (and so Φ) must be unchanged. Use that restriction to argue that λ must be an integer (n).
5. Now look at the differential equation for R, with k set equal to n^2 . Solve that equation.
6. Finally write down the solution for V.