Phys 3330 Electricity & Magnetism II
Spring 2019
Final Exam – Due Monday May 20th, 2019

---

1. A rectangular box has three faces (x=0, y=0, and z=0) held at ground potential, and the other three faces (x=a, y=a, and z=2a) held at a constant potentials V_o, V_o, and $\frac{1}{2}V_o$ respectively. What is the potential at the point (½ a,½ a,½ a)? (You can use the result of assignment 5 without re-deriving the result, and Maple can help evaluate the sum.)

2. A long hollow tube has a cross section which is the pie shaped wedge formed from the x-z plane, the y-z plane, and the curved surface $x^2 + y^2 = R^2$. The two flat surfaces are held at zero potential, and the curved surface has a potential $V(r,\varphi) = V_o \sin\varphi \cos\varphi$. Find the potential inside the wedge. Extra credit - include a 3D graph of the potential.

3. Two concentric spheres have radii R and 3R. The inner sphere as a potential $2V_oP_3(\cos\theta)$ and the outer sphere a potential $46V_oP_5(\cos\theta)$. Find the potential at all points between the two spheres. Extra credit - include a 3D graph of the potential.

4. A metal sphere of radius R is centred on the origin. There is a charge +q at the point (2R,0,0) and a charge -2q at the point (0,5R,0). Find the induced charge density at a point on the sphere and which also lies on the +z axis. (Take the outside of the sphere to be vacuum.)

5. A parallel plate capcitor has plates of area A at the planes z = ±½d, each parallel to the x-y plane. They have charges +Q and -Q respectively. Neglecting edge effects find the fields as measured by an observer moving in the x direction with a speed v.

6. The electric field of a wave is given by
$$\boldsymbol{E}(\boldsymbol{r},t)=\frac{E_o}{\sqrt{2}}(\boldsymbol{k}-\boldsymbol{i})\sin(\kappa y-\omega t)$$
Find the magnetic field and average Poynting vector.

7. Consider two waves traveling in the same direction but with two slightly different angular frequencies $\omega-\frac{1}{2}\Delta\omega$ and $\omega+\frac{1}{2}\Delta\omega$. Let the fields have the same amplitude and polarization.
   a. Show the sum of the two waves is equivalent to a wave moving with a phase velocity $v_p = \omega/\kappa$ but with an amplitude envelope which moves with a group velocity $v_g = \Delta\omega/\Delta\kappa$.
   b. In the limit that $\Delta\omega\to 0$ the group velocity $v_g\to d\omega/d\kappa$. For waves traveling in a plasma we derived the relationship
   $$\kappa^2=\frac{1}{c^2}\left[\omega^2-\omega_p^2\right]$$
   where ω_p is the plasma frequency. Show that for these waves the group velocity has to be less than the speed of light in a vacuum, as required by relativity.