Phys 3330 Electricity & Magnetism II
Spring 2019
Assignment #5 - due Monday March 4th 2019

Laplace's Equation in three variables
Cartesian coordinates

Question

1. An empty 3D box has sides a, b, and c. Five faces of the box are held at zero potential, but the sixth side (z=c) has a potential V_o.
   a. Assuming a potential V(x,y,z) which can be separated, first separate out the term in Z by putting $V(x,y,z) = V_1(x,y)Z(z)$.
   b. Separate the x and y factors by putting $V_1(x,y) = X(x)Y(y)$. (Note: you will have two separation constants at this points, μ and λ.)
   c. Solve each of the ODE´s, and write down an expression for the potential at all points in the box. This potential will contain μ, λ, and an infinite number of arbitrary coefficients.
   d. Apply the boundary conditions to allowed values of μ and λ.
   e. Find the arbitrary coefficients similar to the 2D problem that we covered in class, except that you will now have to integrate over both x from 0 to a, and y from 0 to b.
   f. Write down the final expression for the potential at all points in the box.

$$V(x,y,z) = \frac{16 V_o}{\pi^2} \sum_{n=odd}^{\infty} \sum_{m=odd}^{\infty} \frac{1}{nm} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \frac{\sinh(\gamma z)}{\sinh(\gamma c)}$$