

Relativity

1. Define the transformation matrix by $\mathbf{r}'_4 = \mathbf{L} \mathbf{r}_4$ and

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i \frac{\gamma v}{c} \\ 0 & 0 & -i \frac{\gamma v}{c} & \gamma \end{bmatrix}$$

and the reverse transformation matrix \mathbf{L}' which is the same as \mathbf{L} but with the opposite sign for v . Show that $\mathbf{L}\mathbf{L}' = \mathbf{1}$, the unit matrix which has 1's along the diagonal, and 0's elsewhere.

2. Find the determinant of the matrix \mathbf{L} .
3. Show that $dx' dy' dz' d(ict)' = dx dy dz d(ict)$. Hint: evaluate the [Jacobian](#). This shows that not only do the two observers agree on the 4 dimensional radius they also agree on a 4 dimensional volume.
4. Starting with the formula for the velocity transformation, show that if O is looking at a photon moving with a speed $u = c$, then O' also measure the speed as $u' = c$.
5. Relative to the Earth an object is moving with a speed $u < c$. Starting with the velocity transformation, can a moving observer ($v \neq 0$) ever measure the speed of the object as being greater than the speed of light ($u' > c$)?
6. You are traveling in a spacecraft which is moving with a velocity relative to the Earth equal to $\frac{3}{4}c$ parallel to the ground below you. You fire a missile with a speed of $\frac{1}{2}c$ relative to the spacecraft. What is the speed of the missile relative to the Earth
 - a. If you fire the missile in the same direction as you are moving?
 - b. If you fire the missile in the opposite direction to the direction you are moving?
 - c. If you fire the missile perpendicular to the direction you are moving?