Phys 3330 Electricity & Magnetism II
Spring 2017
Assignment #6 - due Friday March 17th 2017

Image charges

1. For the first image charge problem that we did in class, of a charge located near a metal plate, from the electric field at the plate calculate the surface charge density, and by direct integration find the total charge on the surface of the plate.
   a. With the charge at the point (0,0,d) and a point on the metal plate (z=0) at the point (x,y,z), the distance between the charge and a point on the plate is given by $(x^2+y^2+d^2)^{1/2}$.
   b. The image charge is at the point (0,0,-d) and distance from the point on the metal plate to the image charge is also given by $(x^2+y^2+d^2)^{1/2}$.
   c. At the point on the plate the field is given by the sum of the Coulombic fields from the charge and its image
   $$\boldsymbol{E}=-\frac{1}{2\pi\epsilon_o}\frac{Qd}{(x^2+y^2+d^2)^{3/2}}\boldsymbol{k}$$
   d. From Gauss' Law at the surface of a conductor the surface charge density is $\sigma = \varepsilon_o E$, and the total charge is the integral of the surface charge over the surface of the metal plate, that is the entire x-y plane. Switching to polar coordinates in order to evaluate the integral
   $$Q_{plate}=\int \sigma r\,dr\,d\varphi=-\epsilon_o\frac{1}{2\pi\epsilon_o}Qd\int_{r=0}^{\infty}\frac{r}{(r^2+d^2)^{3/2}}dr\int_{\varphi=0}^{2\pi}d\varphi=-\frac{1}{2\pi}Qd\frac{1}{d}2\pi$$
   e. and so Q_{plate} = -Q.

2. A conductor has a spherical cavity of radius R inside it. Inside this sphere there is a charge +Q located at a distance d from its centre, where d<R. Find the potential inside the cavity, and the charge density on its walls.

3. A charge +Q at the origin is located between two metal plates which occupy the planes z=+a and z=-a.
   a. Find the images charges. (Hint: imagine you are looking at your reflection when standing between two parallel mirrors. What do you see? Looking at it a different way, each image charge has its own image charge.)
   b. Find an expression for the potential at points along the z axis and between the two plates.
   c. Plot the answer to the previous question.
   d. (Optional) Can you plot the potential V(x,0,z) and check that the two plates are equipotentials? (This is a good check that you have the answer to question 3a correct.)

Complex functions

4. For each of the following analytic functions find the equipotentials and electric field lines, and describe the physical situation(s) that relate to it.
   a. f(z) = z
      i. Re(f) = x
      ii. Im(f) = y

iii. If the Re(f) =constant are the equipotentials, then they are planes , and Im(f) = constant are the field line which are parallel to the x axis.
iv. The field is a uniform field $\mathbf{E} = E_o\, \mathbf{i}$

b. $f(z) = \ln(z) = \ln(re^{i\varphi}) = r + i\varphi$
   i. Re(f) = r
   ii. Im(f) = φ
   iii. For a physical application, set Re(f) = constant to be the equipotentials, which are then circles surrounding the origin (cylinders in three dimensions). Im(f) = constant are then the field lines, radiating outward from the origin (from the z axis in three dimensions).
   iv. This is the field produced by a uniform line charge along the z axis.