

Laplace's Equation in three variables
Cartesian coordinates

1. An empty 3D box has sides a , b , and c . Five faces of the box are held at zero potential, but the sixth side ($z=c$) has a potential V_0 .
 - a. Assuming a potential $V(x,y,z)$ which can be separated, first separate out the term in Z by putting $V(x,y,z) = V_1(x,y)Z(z)$.
 - b. Separate the x and y factors by putting $V_1(x,y) = X(x)Y(y)$. (Note: you will have two separation constants at this point, μ and λ .)
 - c. Solve each of the ODE's, and write down an expression for the potential at all points in the box. This potential will contain μ , λ , and an infinite number of arbitrary coefficients.
 - d. Apply the boundary conditions to allowed values of μ and λ .
 - e. Find the arbitrary coefficients similar to the 2D problem that we covered in class, except that you will now have to integrate over both x from 0 to a , and y from 0 to b .
 - f. Write down the final expression for the potential at all points in the box.