

Laplace's Equation in one variable

1. Write down Laplace's equation in Cartesian coordinates if the field is known to be only dependent on one of the coordinates. Show that the field must be uniform.
2. Write down Laplace's equation in spherical polar coordinates if the field is known to be only dependent on the radius (r). Show that the field must be Coulombic.
3. Write down Poisson's equation in Cartesian coordinates if the field is known to be only dependent on one of the coordinates (z). This equation is to be applied to the space between two electrodes when there is a current between them. (In tube electronics this is the diode)
 - a. If the space between the electrodes is evacuated then the electrons do not collide with anything, and are therefore continuously accelerated. That allows us to write the speed of the electrons (v_a) in terms of the potential, $V(z)$.
 - b. The charge density (or the number density of electrons) is not uniform in the space between the electrodes. However, Kirchoff's Law states that the current density (J) must be the same for all values of z . Use the expression $J = n(-e)v_a = -\rho v_a$ to write the charge density as a function of z .
 - c. Use the result of part b to replace the charge density in Poisson's Equation, to get an ordinary differential equation in $V(z)$. Integrate this equation once. (Hint: to integrate manually first multiply each side of the equation by $2dV/dz$, or use Maple.)
 - d. Assuming $dV/dz = 0$ when $z = 0$, integrate once more and show that the current density is inversely proportional to d^2 , and directly proportional to $V_a^{3/2}$, where d is the electrode separation and V_a is the applied voltage.