In the following L is the transformation matrix

$$\boldsymbol{L}=\begin{bmatrix} \gamma & 0 & 0 & i\gamma\frac{v}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\frac{v}{c} & 1 & 0 & \gamma \end{bmatrix}$$

in which

$$\gamma=\frac{1}{\sqrt{1-v^2/c^2}}$$

1. If **L'** is the reverse transformation matrix (replace v with -v) show that **L L'** = 1 (the 4x4 unit matrix)

2. Find the determinant of the matrix **L**.

3. Show that dx' dy' dz' d(ict)' = dx dy dz d(ict). Hint: evaluate the Jacobian

4. Velocity Transformations. With the usual definitions of the components of a velocity vector (u_x = dx/dt, etc) show that

$$\begin{bmatrix} u'_x \\ u'_y \\ u'_z \end{bmatrix}=\frac{1}{1-\frac{u_x v}{c^2}}\begin{bmatrix} u_x-v \\ \frac{u_y}{\gamma} \\ \frac{u_z}{\gamma} \end{bmatrix}$$

   (Stellar Aberration) A beam of light is traveling in the x-y plane (that is v_z=0) in a direction making an angle θ with the x axis, as measured by the stationary observer. However the moving observer with think it is moving in a different direction φ relative to the x axis. Find φ in terms of θ, v, and u_x.

5. Define a 4-vector for the momentum as $\mathbf{p_4}$ = {p_x, p_y, p_z, iE/c}, where E is the energy.
   a. Find the momentum 4-vector for the moving observer defined as $\mathbf{p_4'} = \mathbf{L}\ \mathbf{p_4}$.
   b. Show that $p^2 - E^2/c^2 = p'^2 - E'^2/c^2$. What does each one equal? Why must these expressions be equal.

6. Define a 4-vector for the current as $\mathbf{J_4}$ = {J_x, J_y, J_z, iρ/c}, where ρ is the charge density.
   a. Find the current 4-vector for the moving observer defined as $\mathbf{J_4'} = \mathbf{L}\ \mathbf{J_4}$.
   b. Show that $J^2 - \rho^2/c^2 = J'^2 - \rho'^2/c^2$.
   c. What is $\Sigma\ \partial J_n/\partial x_n$, where n=1..4 is the index of the terms in **J**?
   d. Is this the same relationship for both observers?

7. Define a 4-vector for the potentials as $\mathbf{A_4}$ = {A_x, A_y, A_z, icV}

a. Find the potential 4-vector for the moving observer defined as $\mathbf{A_4'} = \mathbf{L}\ \mathbf{A_4}$.
b. Show that $A^2 - c^2V^2 = A'^2 - c^2V'^2$.
c. What is $\Sigma\ \partial A_n/\partial x_n$, where n=1..4 is the index of the terms in **A**?

8. Given the potential $\mathbf{A'_4}$, find the magnetic field **B'** in terms of **E**, **B**, and the relative velocity of the two observers $\mathbf{v} = v\ \mathbf{i}$.

9. Show that $E^2 - c^2B^2 = E'^2 - c^2B'^2$.

10. Show that **E.B = E'.B'**

11.