Worksheet for Electromagnetic Waves

Plane Waves in Linear Dielectrics

1. Assume E = E(z,t), but with all three Cartesian components. Show that
   a. E_z and B_z must both be constants.
   b. $\partial E_y/\partial z = \partial B_x/\partial t$ and $\partial B_x/\partial z = -\mu\varepsilon\, \partial E_y/\partial t$
   c. $\partial E_x/\partial z = \partial B_y/\partial t$ and $\partial B_y/\partial z = -\mu\varepsilon\, \partial E_x/\partial t$

2. Starting with the equations in part c of the previous problem, show that the electric field E_x obeys the wave equation

$$\frac{d^2 E_x}{dz^2} = \mu\epsilon \frac{d^2 E_x}{dt^2}$$

3. From Electricity and Magnetism I we know that a general solution to the wave equation is $E_x = f(z-vt)$, where $v^2 = \mu\varepsilon$, and f is any function. The simplest solution is a sinusoidal function, $E_x = E_{xo} \cos(\kappa z - \omega t + \varphi)$.
   a. What is the physical significance of κ?
   b. What is the physical significance of ω?
   c. Show that $v = \omega/\kappa = f\lambda$, where f is the frequency and λ is the wavelength.

4. Use the equations from question 1 part c to find B_y.
   a. Show that $B_{yo} = E_{xo}/v$.
   b. Write down expressions for the electric field (E_x) and magnetic field (B_y) for a plane light wave traveling in a vacuum if the electric field amplitude is 250 V/m and the wavelength is 500 nm. Set $\varphi = 0$.

5. Repeat question 2 for the pair of fields E_y and B_x. Are they related to the (E_x,B_y} pair?

6. Show that for an electromagnetic wave traveling in a linear isotropic medium the energy stored in the electric field and the energy stored in the magnetic field each contribute one half of the total stored energy. Hint: we had the expressions in class for a plane wave. Can you show it to be true for any geometry of the wave?

7. The electric field of an electromagnetic wave is given by $E_x = E_z = 0$, $E_y = 320 \cos(3\text{x}10^{11}z - 2500t)$. Find
   a. The magnetic field, including direction.
   b. The frequency.
   c. The wavelength.
   d. The speed of the wave.
   e. The direction of energy flow (propagation direction).
   f. The rate at which energy is transported.

8. The electric field of an electromagnetic wave is given by $E_x = 0$, $E_y = 320 \cos(3x10^{11}z - 2500t)$, and $E_z = = 450 \cos(3x10^{11}z - 2500t)$. Find
   a. The magnetic field, including direction.
   b. The direction of energy flow (propagation direction).
   c. The rate at which energy is transported.

Complex Notation

9. An equivalent complex notation for the electric field is $\mathcal{E} = \mathcal{E}_o e^{i(\boldsymbol{\kappa}.\mathbf{r}-\omega t)}$, where $\mathcal{E} = \mathbf{E_o} e^{i\varphi}$, is the complex amplitude, including all the phase information. The vector $\boldsymbol{\kappa}$ is the known as the propagation vector, and **r** is the position vector. Show that
   a. div $\mathcal{E} = i\, \boldsymbol{\kappa} . \mathcal{E}$
   b. curl $\mathcal{E} = i\, \boldsymbol{\kappa} \times \mathcal{E}$

10. Show that in complex notation Maxwell's Equations can be written as
   a. $\boldsymbol{\kappa} . \mathcal{E} = 0$
   b. $\boldsymbol{\kappa} . \mathcal{B} = 0$
   c. $\boldsymbol{\kappa} \times \mathcal{E} = \omega\, \mathcal{B}$
   d. $\boldsymbol{\kappa} \times \mathcal{B} = \mu\varepsilon\omega\, \mathcal{E}$

11. In the previous problem, separate variables by taking the cross product of both sides of the third equation with κ, and show that $\kappa^2 = \mu\varepsilon\omega^2$.

12. The Poynting vector is $<S> = ½\, E_o^2/\mu v$, where E_o is the amplitude of the ***real*** electric field. In complex notation the real part of a complex number (z) can always be written as $½(z+z^*)$. Show that in complex notation that $<S> = ½\, |\mathcal{E}|^2 / \mu v = ½\, \mathcal{E}. \mathcal{E}^*/\mu v$.

Spherical waves

13. Starting with the wave equation, look for a spherically symmetric solution $E = E(r,t)$ with no θ or φ dependence. Hint: Use the substitution $E(r,t) = 1/r\, f(r,t)$ to find a general solution. What physical interpretation can you give to each part of the solution?

14. Starting with the solution from the previous problem, calculate the Poynting vector. It should have a $1/r^2$ dependence. Can you interpret that?

Reflection and Transmission - dielectrics

15. Light of wavelength λ is incident normally on an infinitely thick slab of a dielectric whose dielectric constant is K_e. Find the intensity reflection and transmission coefficients.

16. In the previous problem, show that the energy carried in by the incident wave is exactly equal to the energy carried away by the reflected and refracted waves.

17. Light of wavelength λ is incident normally on a sheet of a dielectric of thickness d and dielectric constant K_e. Find the intensity refection and transmission coefficients. (Note that this problem is identical to problem you might have solved in Quantum Mechanics, for a particle of energy E passing over a “wall” whose height is less than E.)

18. In the previous problem, show that the energy carried in by the incident wave is exactly equal to the energy carried away by the reflected and refracted waves.

Reflection and Transmission – Metals

19. Show that when monochromatic light is incident normally on a perfect conductor there is a $\pi/4$ phase difference between the electric and magnetic field vectors.

20. Show that when monochromatic light is incident normally on a perfect conductor the reflection coefficient is identically equal to 1.