Phys 3320 Electricity & Magnetism I
Fall 2018
Mid Term - December 7th, 2018

Section 1

1. Write down all four of Maxwell's Equations in their most general form.

   $\text{div}\, \boldsymbol{D} = \rho$

   $\text{div}\, \boldsymbol{B} = 0$

   $\text{curl}\, \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$

   $\text{curl}\, \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

   Although the four Maxwell's equations are presented as independent, in fact there are interrelationships.

   a) show that the first and fourth equation lead to the continuity equation, div **J** = - ∂ρ/∂t.

   $$\text{div curl}\, \boldsymbol{H} = \text{div}\, \boldsymbol{J} + \text{div} \frac{\partial \boldsymbol{D}}{\partial t} = \text{div}\, \boldsymbol{J} + \frac{\partial\, \text{div}\, \boldsymbol{D}}{\partial t}$$

   $$0 = \text{div}\, \boldsymbol{J} + \frac{\partial \rho}{\partial t}$$

   b) show that the third equation leads to the result that div **B** is time independent, consistent with the second equation.

   $$\text{div curl}\, \boldsymbol{E} = -\,\text{div} \frac{\partial \boldsymbol{B}}{\partial t} = -\frac{\partial\, \text{div}\, \boldsymbol{B}}{\partial t}$$

   $$0 = \frac{\partial\, \text{div}\, \boldsymbol{B}}{\partial t}$$

   $$\text{div}\, \boldsymbol{B} = \text{constant}$$

2. Is a magnetostatic field conservative? Give a brief explanation. No. For a conservative field the curl must be zero. In the case of the magnetostatic field the curl is not in general 0.

3. *In the general case,* define the displacement and induction fields. What are the advantages in working with these fields rather than the electric and magnetic fields?
   a) $\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}$
   b) $\mathbf{H} = \mathbf{B}/\mu_o - \mathbf{M}$
   c) In the case of the displacement field it depends only on the free charges. Similarly, the induction field only depends on free currents.

4. What are the physical reasons why some materials are diamagnetic, some are paramagnetic, and others are ferromagnetic?
   a) Diamagnetic – the atoms which make up the material have no existing magnetic dipole moments.
   b) Paramagnetic - the atoms which make up the material do have existing magnetic dipole moments, which are randomly oriented. When an external field is applied they act independently of one another.
   c) Ferromagnetic - the atoms which make up the material do have existing magnetic dipole moments, which are not randomly oriented. Instead they are strongly correlated, with all

dipole moments within a domain having the same direction. When an external field is applied they act cohesively.

Section 2

5. A cylinder of radius R is made from a ferromagnetic material with a permanent magnetization **M** = M_o s^2 **φ** where s is the distance from the axis. Find the magnetic field both inside and outside the cylinder. (Hint: think bound currents and Ampere's law.)

6. Write down Maxwell's 4th equation for the electromagnetic fields in a LIH material, and substitute for the fields in terms of the potentials V and **A**.
   a) Show that the equations reduce to the wave equation with a source term

$$\nabla^2 \boldsymbol{A} - \mu\epsilon\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J}$$

   where $\varepsilon = \varepsilon_r\varepsilon_o$ and $\mu = \mu_r\mu_o$ providing the correct gauge is chosen, and find the condition that this gauge must satisfy.
   b) Find the equivalent wave equation for the scalar potential V in this gauge.

7. Two concentric metal spheres have radii R and 4R. Between R and 3R there is a dielectric of relative permitivitty ε_1, and between 3R and 4R there is a dielectric of relative permitivitty ε_2. There is also a charge +Q on the inner sphere, and a charge -Q on the outer sphere.
   a) Find the energy density at all points, and from that the total stored energy.
   b) From the total stored energy deduce the capacitance of the system.